

## Questions on Lesson 4.4?

We'll go over any questions you have, in the meantime, solve this problem.

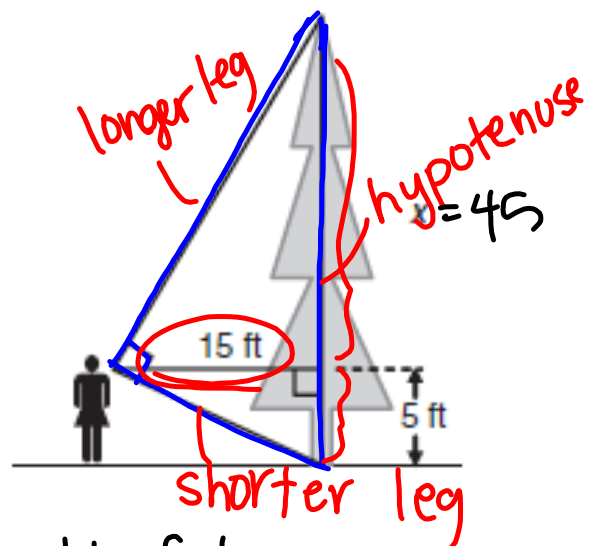
You are standing 15 feet from a tree. Your line of sight to the top of the tree and to the bottom of the tree forms a 90-degree angle as shown in the diagram. The distance between your line of sight and the ground is 5 feet. Estimate the height of the tree.

$$\frac{15}{x} = \frac{5}{15}$$

$$5x = \frac{225}{5}$$

$$x = 45$$

$$\frac{x}{15} = \frac{15}{5}$$



Height of tree:

$$45 + 5 = 50 \text{ ft.}$$

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2. In each triangle, solve for x.

a.

shorter leg

4

hypotenuse

9

altitude

x

longer leg

$x$

$x$

$2x$

$x$

$x^2$

$$\frac{4}{x} = \frac{x}{9}$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = 6$$

$$\frac{x}{4} = \frac{9}{x}$$

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b.

4

x

8

altitude

$$\frac{8}{4} = \frac{x}{8}$$
$$\frac{4x}{4} = \frac{64}{4}$$
$$\boxed{x = 16}$$
$$\frac{4}{8} = \frac{8}{x}$$

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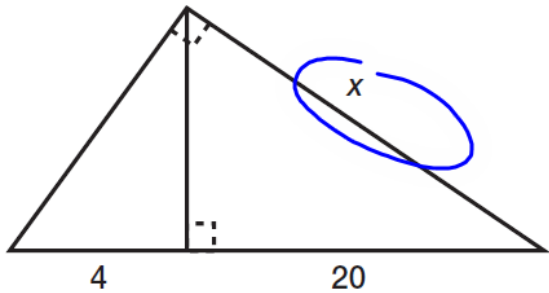
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C.



$\frac{x}{24} = \frac{20}{x}$

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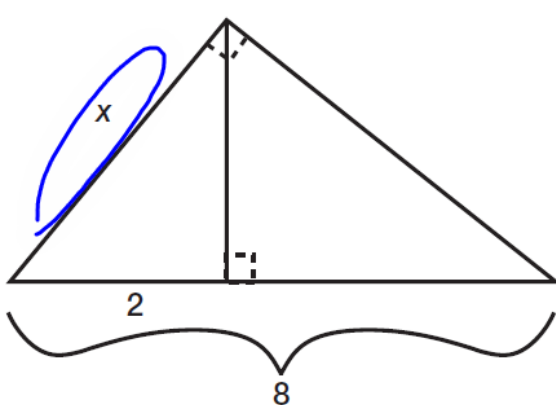
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d.


$$\frac{x}{8} = \frac{2}{x}$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = 4$$

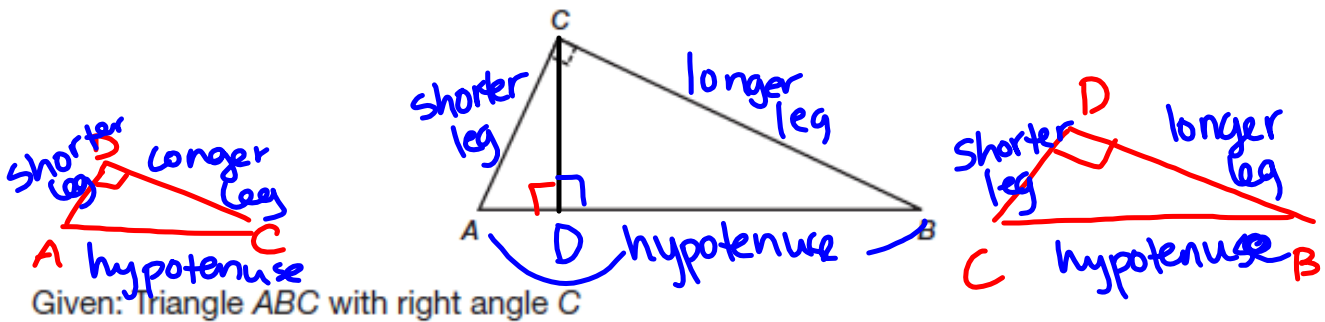
# Proving the Pythagorean Theorem

4.5

## Proving the Pythagorean Theorem and the Converse of the Pythagorean Theorem

PG.312 IN YOUR BOOK

Use the Right Triangle/Altitude Similarity Theorem to prove the Pythagorean Theorem.



Given: Triangle ABC with right angle C

Prove:  $AC^2 + CB^2 = AB^2$

1. Construct altitude  $CD$  to hypotenuse  $AB$ .
2. Applying the Right Triangle/Altitude Similarity Theorem, what can you conclude?

All 3  $\Delta$ s are  $\sim$ , so

$$\Delta ABC \sim \Delta CBD \sim \Delta ACD$$

3. Write a proportional statement describing the relationship between the longest leg and hypotenuse of triangle  $ABC$  and triangle  $CBD$ .

$$\frac{\Delta ABC}{\Delta CBD} \rightarrow \frac{AB}{CB} = \frac{CB}{DB}$$

4. Rewrite the proportional statement you wrote in Question 3 as a product.

$$(AB)(DB) = (CB)(CB)$$

$$(AB)(DB) = (CB)^2$$

5. Write a proportional statement describing the relationship between the shortest leg and hypotenuse of triangle  $ABC$  and triangle  $ACD$ .

$$\frac{\Delta ABC}{\Delta ACD} \rightarrow \frac{AC}{AD} = \frac{AB}{AC}$$

$$(6) (AB)(AD) = (AC)^2$$

## PG.313 IN YOUR BOOK

6. Rewrite the proportional statement you wrote in Question 5 as a product.

$$(AC)^2 = (AB)(AD)$$

7. Add the statement in Question 4 to the statement in Question 6.

$$(CB)^2 = (AB)(DB)$$

$$+ (AC)^2 = (AB)(AD)$$

---


$$(AC)^2 + (CB)^2 = (AB)(DB) + (AB)(AD)$$

8. Factor the statement in Question 7.

$$(AC)^2 + (CB)^2 = (AB)(DB + AD)$$

$$2x + 2y$$

$$2(x + y)$$

9. What is equivalent to  $DB + AD$ ?

$$(AB)$$

10. Substitute the answer to Question 9 into the answer to Question 8 to prove the Pythagorean Theorem.

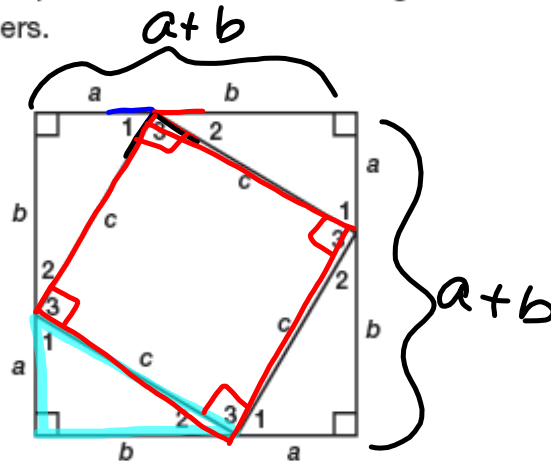
$$(AC)^2 + (CB)^2 = (AB)(AB)$$

$$(AC)^2 + (CB)^2 = (AB)^2$$

## PG.315 IN YOUR BOOK

Use the diagram shown and the following questions to prove the Converse of the Pythagorean Theorem.

A large square is composed with four identical right triangles in its corners.



Recall, the Converse of the Pythagorean Theorem states: "If  $a^2 + b^2 = c^2$ , then  $a$  and  $b$  are the lengths of the legs of a right triangle and  $c$  is the length of the hypotenuse."



1. What can you conclude about  $m\angle 1 + m\angle 2 + m\angle 3$ ?

$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

2. Use the Triangle Sum Theorem to determine  $m\angle 1 + m\angle 2$ .

$$\begin{array}{r} m\angle 1 + m\angle 2 + 90^\circ = 180^\circ \\ -90 \quad -90 \\ \hline m\angle 1 + m\angle 2 = 90^\circ \end{array}$$

3. Knowing  $m\angle 1 + m\angle 2$ , what can you conclude about  $m\angle 3$ ?

$$m\angle 3 = 90^\circ$$

4. What does  $m\angle 3$  tell you about the quadrilateral inside of the large square?

square

5. What is the area of one of the right triangles?

$$A_{\Delta} = \frac{ab}{2}$$



## PG.316 IN YOUR BOOK

6. What is the area of the quadrilateral inside the large square?

$$A_D = c^2$$

7. Write an expression that represents the combined areas of the four right triangles and the quadrilateral inside the large square. ~~Use your answers from Question 6, parts (c) and (d).~~

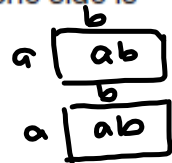
$$A_{\text{total}} = 4\left(\frac{ab}{2}\right) + c^2$$

$$A_{\text{total}} = \frac{4ab}{2} + c^2$$

$$A_{\text{total}} = 2ab + c^2$$

8. Write an expression to represent the area of the large square, given that one side is expressed as  $(a + b)$ . Simplify your answer.

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2$$



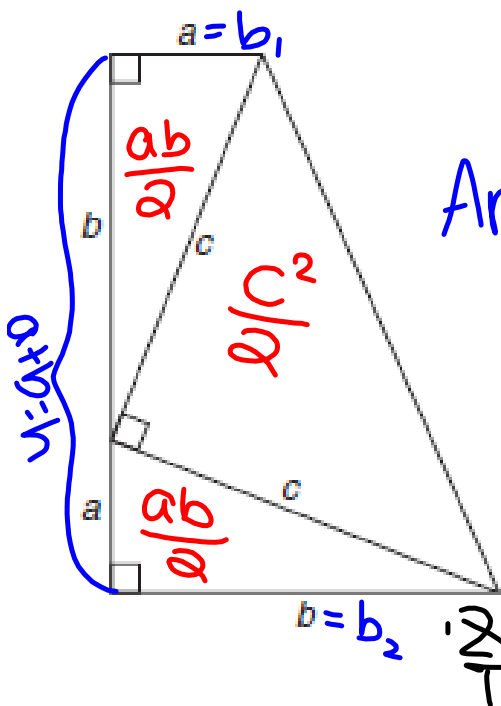
$$A_{\text{total}} = a^2 + 2ab + b^2$$

9. Write an equation using the two different expressions representing the area of the large square from Questions 7 and 8. Then, solve the equation to prove the Converse of the Pythagorean Theorem.

$$\begin{array}{r}
 2ab + c^2 = a^2 + 2ab + b^2 \\
 - 2ab \qquad \qquad - 2ab \\
 \hline
 c^2 = a^2 + b^2
 \end{array}$$

NOT IN YOUR BOOK

In order to prove the Pythagorean Theorem using this figure, show that the sum of the three triangles is equal to the area of the trapezoid. (Note:  $A_{\text{trapezoid}} = h \left( \frac{b_1 + b_2}{2} \right)$  where  $h$  is the height and  $b$  is the base.)



$$A_{\text{total}} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$$

$$\text{Area}_{\text{trapezoid}} = (a+b) \left( \frac{a+b}{2} \right)$$

$$\frac{(a+b)(a+b)}{2} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$$

$$\frac{(a+b)(a+b)}{2} = \frac{ab+ab+c^2}{2}$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\frac{a^2 + 2ab + b^2 - 2ab}{1} = \frac{2ab + c^2 - 2ab}{1}$$

$$a^2 + b^2 = c^2$$

# Homework

## Finish 4.5