Making Inferences and Justifying Conclusions

Every 2,

Y, and 6 years, Americans head to the polls to select the women and men who will represent them in the Congress and the White House. And more often than that, these Americans will be polled about their choices. Although election polls can be remarkably accurate, there is always some margin for error.

> **For Real?** 2.1 Sample Surveys, Observational Studies, 2.2 **Circle Up** Sampling Methods and Randomization57 2.3 **Sleep Tight** Using Confidence Intervals to Estimate 2.4 **How Much Different?** Using Statistical Significance to 2.5 DIY

2

Designing a Study and Analyzing the Results 101

For Real? Sample Surveys, Observational Studies, and Experiments

LEARNING GOALS

In this lesson, you will:

- Identify characteristics of sample surveys, observational studies, and experiments.
- Differentiate between sample surveys, observational studies, and experiments.
- Identify possible confounds in the design of experiments.

KEY TERMS

- characteristic of interest
- sample survey
- random sample
- biased sample
- observational study
- experiment
- treatment
- experimental unit
- confounding

Have you taken medicine to treat an illness? Imagine that the medicine you took was not really medicine, but just a sugar pill. In medical studies, people who have unknowingly taken a sugar pill—called a placebo—have reported that the pill has had an effect similar to medicine, even though there was no medicine in the pill at all. This is an example of what is called the placebo effect.

Researchers must always be on the lookout for placebo effects. They may be to blame for successful or unsuccessful outcomes to experiments.

PROBLEM 1 Survey Says



You can use data to help answer questions about the world. The specific question that you are trying to answer or the specific information that you are trying to gather is called a **characteristic of interest**.

For example, you can use data to help determine which drug is most effective, teenagers' favorite television program, or how often doctors wash their hands.

One way of collecting data is by using a *sample survey*. A **sample survey** poses one or more questions of interest to obtain sample data from a population. Recall, a population represents all the possible data that are of interest in a survey, and a sample is a subset of data that is selected from the population.

A researcher wants to design a sample survey to determine the amount of time that U.S. teenagers between the ages of 16 to 18 spend online each day.



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- 1. Identify the characteristic of interest in the sample survey.
- 2. Identify the population that the researcher is trying to measure by using a sample survey.

3. Augie and Sandy	were discussing the po	pulation of the survey.	
	Augie The population is all 16- to 18-year-olds in the United States.	Sandy The population is all teenagers in the Urited States.	
Who is correct? E	xplain your reasoning.		



4. Write a survey question or questions that the researcher could use to collect data from the participants in the survey.



When sample data are collected in order to describe a characteristic of interest, it is important that such a sample be as representative of the population as possible. One way to collect a representative sample is by using a *random sample*. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected. A **biased sample** is a sample that is collected in a way that makes it unrepresentative of the population.



5. Joanie and Richie were discussing strategies the researcher could use to select a representative sample of 16- to 18-year-olds.

Joanie

The sample should include 16- to 18-year-olds from various states. Richie

Include 16- to 18-year-olds who live in various types of communities, like urban, suburban, and rural.

List some additional strategies the researcher should consider when selecting the sample.

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6. Cherese suggested that the researcher could post the survey online and then distribute the link to the survey to students after school on Friday as they are leaving the building. Will this method result in a biased sample? Explain your reasoning.

PROBLEM 2 Confound It All!

In an **observational study,** data are gathered about a characteristic of the population by simply observing and describing events in their natural settings. Recording the number of children who use the swings at a local park would be an example of a simple observational study.



The results of an observational study state that approximately 70% of in-house day care centers in one U.S. state show as much as 2.5 hours of television to the children per day. The observational study examined 132 day care centers in one state.

1. Identify the population, the sample, and the characteristic of interest in the observational study.



2. List some similarities and differences between an observational study and a sample survey.



An **experiment** gathers data on the effect of one or more **treatments**, or experimental conditions, on the characteristic of interest. Members of a sample, also known as **experimental units**, are randomly assigned to a treatment group.

Researchers conducted an experiment to test the effectiveness of a new asthma drug. They collected data from a sample of 200 asthma patients. One hundred of the patients received a placebo treatment along with an inhaler. The other one hundred patients received the new drug along with an inhaler. Monthly blood and breathing tests were performed on all 200 patients to determine if the new drug was effective.

3. Identify the population, the sample, and the characteristic of interest in the experiment.



4. What are the treatments in the experiment?

5. What are some ways the researchers could choose a biased sample for this experiment?

Confounding occurs when there are other possible reasons, called confounds, for the results to have occurred that were not identified prior to the study.



6. Suppose one of the treatment groups was given the new drug with an inhaler and the other group was given a placebo with no inhaler. Describe how this design of the experiment introduces a confound.

Talk the Talk



Classify each scenario as a sample survey, an observational study, or an experiment, and explain your reasoning. Then, identify the population, the sample, and the characteristic of interest.

1. To determine whether there is a link between high-voltage power lines and illnesses in children who live in the county, researchers examined the illness rate for 100 children that live within $\frac{1}{4}$ of a mile from power lines and the illness rate for 100 children that live more than $\frac{1}{4}$ of a mile from power lines.

2. Seventy of the school's calculus students are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. The math department team leader wants to determine whether there is a link between graphing calculator use and students' calculus grades. **3.** A medical researcher wants to learn whether or not there is a link between the amount of TV children watch each day and childhood obesity in a particular school district. She gathers data from the records of 15 local pediatricians.

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4. In a particular school district, a researcher wants to learn whether or not there is a link between a child's daily amount of physical activity and their overall energy level. During lunch at a school, she distributed a short questionnaire to students in the cafeteria.

Online Time Study, Part I



To design a sample survey, observational study, or experiment, consider these steps:

- Identify the characteristic of interest.
- Identify the population.
- Identify methods to collect the sample so that the sample is not biased.
- Ensure that participants are randomly assigned to a treatment.
- Eliminate elements of the design that may introduce confounding.
- 29
- Design a data collection plan to learn how much time students in your school spend online each day.
 - **a.** Identify the population and the characteristic of interest.



b. Is the most efficient method for collecting the data a sample survey, an observational study, or an experiment? Explain your reasoning.

c. Explain how you can gather data from a representative, unbiased sample of students in your school.



Be prepared to share your solutions and methods.

Circle Up Sampling Methods and Randomization

LEARNING GOALS

In this lesson, you will:

- Use a variety of sampling methods to collect data.
- Identify factors of sampling methods that could contribute to gathering biased data.
- Explore, identify, and interpret the role of randomization in sampling.
- Use data from samples to estimate population mean.

KEY TERMS

- convenience sample
- subjective sample
- volunteer sample
- simple random sample
- stratified random sample
- cluster sample
- cluster
- systematic sample
- parameter
- statistic

When you play word games like this, where you guess the letters until you figure out the word, you think about samples and populations.

For example, you know that the missing word is a sample of the population of words in the English language. Since "e" is a frequently used letter and "z" is used infrequently in words, you would probably guess "e" before you guessed "z".

It is useful in statistics, too, to assume that the characteristics of a sample match those of a population—as long as that sample is chosen wisely!

PROBLEM 1 You Gotta Mix It Up



When you use statistics, you are often measuring the values of a population by focusing on the measurements of a sample of that population. A population does not have to refer to people. It can be any complete group of data—like the areas of 100 circles.

The end of this lesson includes 100 circles and a table. The table lists an identification number, the diameter, and the area for each circle. Suppose you want to determine the mean area of all 100 circles. Calculating the areas of all of the circles would be time-consuming. Instead, you can use different samples of this population of circles to estimate the mean area of the entire population.

 Without looking at the circles, Mauricia decided to use Circles 1–5 for her sample. Is it likely that those 5 circle areas are representative of all 100 circles? Explain your reasoning.



 Analyze the circles. Select a sample of 5 circles that you think best represents the entire set of circles.

The sample of circles Mauricia chose is called a *convenience sample*. A **convenience sample** is a sample whose data is based on what is convenient for the person choosing the sample.

The sample of circles you chose in Question 2 is called a *subjective sample*. A **subjective sample** is a sample drawn by making a judgment about which data items to select.

Another type of sample is a *volunteer sample*. A **volunteer sample** is a sample whose data consists of those who volunteer to be part of a sample.



3. Olivia and Ricky discussed whether a convenience sample or a subjective sample is more likely to be representative of the population of circle areas.

Olivia

I think a subjective sample is more likely to be representative of the 100 circles than the convenience sample.

Ricky

The subjective sample and the convenience sample are equally likely to be representative of the 100 circles.

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Who is correct? Explain your reasoning.

 Olivia shared her conclusion about convenience samples, subjective samples, and volunteer samples.

Olivia

Even though one method may be better than another in a specific situation, collecting data using a convenience sample, subjective sample, or volunteer sample will likely result in a biased sample.





Explain why Olivia's statement is correct.

PROBLEM 2 Equal Opportunity for All



A **simple random sample** is a sample composed of data elements that were equally likely to have been chosen from the population.

1. Explain how convenience samples, subjective samples, and volunteer samples do not include data elements that were equally likely to have been chosen from the population.

Using a random digit table is one option for selecting a simple random sample. To use the table, begin at any digit and follow the numbers in a systematic way, such as moving across a row until it ends and then moving to the beginning of the next row.

				Rando	om Digit	Table				
Line 1	65285	97198	12138	53010	94601	15838	16805	61004	43516	17020
Line 2	17264	57327	38224	29301	31381	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86210	11808	12841	45147	97438	60022	12645	62000
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

You can use two digits at a time to choose a sample of 5 circles.



2. Select a simple random sample of 5 circles using the random digit table. Pick any row of the table. Use the first two digits to represent the first circle of the sample, the next two digits to represent the second circle of the sample, and so on. List the identification numbers of the 5 circles.



You can also use a graphing calculator to generate a random list of numbers.



- **3.** Use a graphing calculator to generate a random sample of 5 circles.
- 4. Calculate the mean area of the circles in your simple random sample.



5. Compare your simple random sample with your classmates' samples. What do you notice?



There are several other types of random samples, including *stratified random samples*, *cluster samples*, and *systematic samples*.

A **stratified random sample** is a random sample obtained by dividing a population into different groups, or strata, according to a characteristic and randomly selecting data from each group.

You can collect a stratified random sample of circles by first dividing the circles into groups.

	B			
ϵ	R	Define groups of circles ba	ased on the lengths of their	diameters.
		 Small circles: diameter 	$\leq \frac{1}{4}$ in.	
	B	• Medium circles: $\frac{1}{2}$ in. <	diameter $\leq 1\frac{1}{2}$ in.	
\in	B	Large circles: diameter	$> 1\frac{1}{2}$ in.	
	P	Small Circles (46)	Medium Circles (39)	Large Circles (15)
\in	B	1, 4, 6, 13, 14, 16, 17, 19, 22, 24, 26, 28, 30, 33, 34,	0, 2, 3, 8, 9, 10, 11, 12, 21, 23, 25, 29, 31, 35, 36,	5, 7, 15, 18, 20, 27, 32, 38, 44, 48, 54, 55, 60, 70,
\in	B	37, 39, 42, 45, 46, 47, 51, 53, 56, 57, 58, 59, 62, 63,	40, 41, 43, 49, 50, 52, 61, 64, 65, 66, 69, 71, 73, 75,	92
∈	B	67, 68, 72, 74, 78, 79, 82, 85, 87, 88, 89, 93, 94, 95,	76, 77, 80, 81, 83, 84, 86, 90, 91, 96	
	B	97, 98, 99		
\in	B	There are about an equal r	number of small and	Another option
		medium circles and about	a third as many	is to randomly select 2
	3	small circles. To maintain t	his ratio in your	large circles, 6 medium circles
\leq	3	stratified random sample,	you can choose	and 6 small circles. This keep
	-	3 small circles, 3 medium (circles, and	The varios the same.
	B	i sinali circie.		
\in	P	Select random circles from random digit table or a gra	n each group using a phing calculator.	1001
\in	P			E,



6. Collect a stratified random sample of circles. List the sample and explain your method.

A **cluster sample** is a random sample that is obtained by creating *clusters*. Then, one cluster is randomly selected for the sample. Each **cluster** contains the characteristics of a population.

- **8.** Use the page that contains the circles at the end of this lesson to answer each question.
 - a. Draw 4 horizontal lines and 2 vertical lines so that the page is divided into 12 congruent rectangles. Each rectangle represents a cluster of circles. Number each cluster from 1 to 12.



 b. Use a graphing calculator or the random digit table to randomly select one of the clusters. List the cluster sample.



c. Calculate the mean of the circle areas included in your cluster sample.

A **systematic sample** is a random sample obtained by selecting every *n*th data value in a population.



9. Select a systematic sample by choosing every 20th circle. First, randomly choose a number from 0 to 20 to start at and then choose every 20th circle after that.



10. Calculate the mean of the circle areas included in your systematic sample.



11. Faheem and Calvin shared their thoughts about random sampling.

Faheem

Simple random sampling, stratified random sampling, and cluster sampling will always produce a representative, unbiased sample.

Calvin

Simple random sampling, stratified random sampling, or cluster sampling does not guarantee a representative, unbiased sample.

Who is correct? Explain your reasoning.

The mean of a sample, \bar{x} , can be used to estimate the population mean, μ . The population mean is an example of a **parameter**, because it is a value that refers to a population. The sample mean is an example of a **statistic**, because it is a value that refers to a sample.

The population mean for the 100 circles is $\mu = 0.58\pi$ square inches, or approximately 1.82 square inches.

	Carla	
_	I didn't expect the sample of 5 circles to have a mean closest to the mean of the population. I must have done something wrong when collecting the samples.	
	Mean of 5 circles \approx 0.55 π square inches	
	Mean of 15 circles \approx 0.49 π square inches	
	Mean of 30 circles \approx 0.65 π square inches	



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Online Time Study, Part II



In the first lesson of this chapter, you designed a plan to learn about the amount of time students in your school are online each day.

1. Which sampling method would be best to select the data? Explain your reasoning.





Be prepared to share your results and methods.



2

Circle Number	Diameter (in.)	Area (in.²)	Circle Number	Diameter (in.)	Area (in.²)	Circle Number	Diameter (in.)	Area (in.²)
0	$\frac{1}{2}$	$\frac{1}{16}\pi$	18	2	π	36	1	$\frac{1}{4}\pi$
1	$\frac{1}{4}$	$\frac{1}{64}\pi$	19	$\frac{1}{4}$	$\frac{1}{64}\pi$	37	$\frac{1}{4}$	$\frac{1}{64}\pi$
2	$\frac{1}{2}$	$\frac{1}{16}\pi$	20	$1\frac{1}{2}$	$\frac{9}{16}\pi$	38	2	π
3	1	$\frac{1}{4}\pi$	21	$\frac{1}{2}$	$\frac{1}{16}\pi$	39	$\frac{1}{4}$	$\frac{1}{64}\pi$
4	$\frac{1}{4}$	$\frac{1}{64}\pi$	22	$\frac{1}{4}$	$\frac{1}{64}\pi$	40	$\frac{1}{2}$	$\frac{1}{16}\pi$
5	$1\frac{1}{2}$	$\frac{9}{16}\pi$	23	1	$\frac{1}{4}\pi$	41	1	$\frac{1}{4}\pi$
6	$\frac{1}{4}$	$\frac{1}{64}\pi$	24	$\frac{1}{4}$	$\frac{1}{64}\pi$	42	$\frac{1}{4}$	$\frac{1}{64}\pi$
7	2	π	25	$\frac{1}{2}$	$\frac{1}{4}\pi$	43	1	$\frac{1}{4}\pi$
8	$\frac{1}{2}$	$\frac{1}{16}\pi$	26	$\frac{1}{4}$	$\frac{1}{64}\pi$	44	2	π
9	1	$\frac{1}{4}\pi$	27	$1\frac{1}{2}$	$\frac{9}{16}\pi$	45	$\frac{1}{4}$	$\frac{1}{64}\pi$
10	1	$\frac{1}{4}\pi$	28	$\frac{1}{4}$	$\frac{1}{64}\pi$	46	$\frac{1}{4}$	$\frac{1}{64}\pi$
11	$\frac{1}{2}$	$\frac{1}{16}\pi$	29	1	$\frac{1}{4}\pi$	47	$\frac{1}{4}$	$\frac{1}{64}\pi$
12	$\frac{1}{2}$	$\frac{1}{16}\pi$	30	$\frac{1}{4}$	$\frac{1}{64}\pi$	48	2	π
13	$\frac{1}{4}$	$\frac{1}{64}\pi$	31	1	$\frac{1}{4}\pi$	49	$\frac{1}{2}$	$\frac{1}{16}\pi$
14	$\frac{1}{4}$	$\frac{1}{64}\pi$	32	2	π	50	$\frac{1}{2}$	$\frac{1}{16}\pi$
15	2	π	33	$\frac{1}{4}$	$\frac{1}{64}\pi$	51	$\frac{1}{4}$	$\frac{1}{64}\pi$
16	$\frac{1}{4}$	$\frac{1}{64}\pi$	34	$\frac{1}{4}$	$\frac{1}{64}\pi$	52	$\frac{1}{2}$	$\frac{1}{64}\pi$
17	$\frac{1}{4}$	$\frac{1}{64}\pi$	35	$\frac{1}{2}$	$\frac{1}{16}\pi$	53	$\frac{1}{4}$	$\frac{1}{64}\pi$

Circle Number	Diameter (in.)	Area (in.²)	Circle Number	Diameter (in.)	Area (in.²)	Circle Number	Diameter (in.)	Area (in.²)
54	$1\frac{1}{2}$	$\frac{9}{16}\pi$	72	$\frac{1}{4}$	$\frac{1}{64}\pi$	90	<u>1</u> 2	$\frac{1}{16}\pi$
55	2	π	73	<u>1</u> 2	$\frac{1}{16}\pi$	91	<u>1</u> 2	$\frac{1}{16}\pi$
56	$\frac{1}{4}$	$\frac{1}{64}\pi$	74	$\frac{1}{4}$	$\frac{1}{64}\pi$	92	2	π
57	$\frac{1}{4}$	$\frac{1}{64}\pi$	75	$\frac{1}{2}$	$\frac{1}{16}\pi$	93	$\frac{1}{4}$	$\frac{1}{64}\pi$
58	$\frac{1}{4}$	$\frac{1}{64}\pi$	76	$\frac{1}{2}$	$\frac{1}{16}\pi$	94	$\frac{1}{4}$	$\frac{1}{64}\pi$
59	$\frac{1}{4}$	$\frac{1}{64}\pi$	77	1	$\frac{1}{4}\pi$	95	$\frac{1}{4}$	$\frac{1}{64}\pi$
60	2	π	78	$\frac{1}{4}$	$\frac{1}{64}\pi$	96	$\frac{1}{2}$	$\frac{1}{16}\pi$
61	$\frac{1}{2}$	$\frac{1}{16}\pi$	79	$\frac{1}{4}$	$\frac{1}{64}\pi$	97	$\frac{1}{4}$	$\frac{1}{64}\pi$
62	$\frac{1}{4}$	$\frac{1}{64}\pi$	80	1	$\frac{1}{4}\pi$	98	$\frac{1}{4}$	$\frac{1}{64}\pi$
63	$\frac{1}{4}$	$\frac{1}{64}\pi$	81	$\frac{1}{2}$	$\frac{1}{16}\pi$	99	$\frac{1}{4}$	$\frac{1}{64}\pi$
64	$\frac{1}{2}$	$\frac{1}{16}\pi$	82	$\frac{1}{4}$	$\frac{1}{64}\pi$			
65	1	$\frac{1}{4}\pi$	83	$\frac{1}{2}$	$\frac{1}{16}\pi$			
66	<u>1</u> 2	$\frac{1}{16}\pi$	84	$\frac{1}{2}$	$\frac{1}{16}\pi$			
67	$\frac{1}{4}$	$\frac{1}{64}\pi$	85	$\frac{1}{4}$	$\frac{1}{64}\pi$			
68	$\frac{1}{4}$	$\frac{1}{64}\pi$	86	1	$\frac{1}{4}\pi$			
69	$\frac{1}{2}$	$\frac{1}{16}\pi$	87	$\frac{1}{4}$	$\frac{1}{64}\pi$			
70	2	π	88	$\frac{1}{4}$	$\frac{1}{64}\pi$			
71	$\frac{1}{2}$	$\frac{1}{16}\pi$	89	$\frac{1}{4}$	$\frac{1}{64}\pi$			

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2.3

Sleep Tight Using Confidence Intervals to Estimate Unknown Population Means

LEARNING GOALS

In this lesson, you will:

- Interpret the margin of error for estimating a population proportion.
- Interpret the margin of error for estimating a population mean.
- Recognize the difference between a sample and a sampling distribution.
- Recognize that data from samples are used to estimate population proportions and population means.
- Use confidence intervals to determine the margin of error of a population proportion estimate.
- Use confidence intervals to determine the margin of error of a population mean estimate.

KEY TERMS

- population proportion
- sample proportion
- sampling distribution
- confidence interval

hy do we have dreams? Scientists still don't really have the answer to that question, but there have been many theories.

Some suggest that dreaming is the brain's way of discarding memories you have gathered during the day but no longer need, and studies have shown that dreaming increases as a result of learning. Another theory suggests that your brain is simply constantly churning out thoughts and images and that this doesn't stop when the rest of your body is asleep.

Some scientists are looking to evolution to provide some clues about why we dream—especially since humans don't seem to be the only animals that dream.

Why do you think some animals dream?

PROBLEM 1 Every Vote Counts: Exploring Categorical Data

In a poll of 1100 registered voters before an upcoming mayoral election, 594 people, or 54%, said they would vote to re-elect the current mayor, while the remaining voters said they would not vote for the mayor. The margin of error for the poll was ± 3 percent, which means that the poll predicts that somewhere between 51% (54% - 3%) and 57% (54% + 3%) of people will actually vote to re-elect the mayor.

1. Does the poll represent a sample survey, an observational study, or an experiment?



2. Based on the poll, can you conclude that the current mayor will be re-elected? Explain your reasoning.



3. Is it possible for fewer than 50% of respondents in a new sample to respond that they will vote for the mayor in the election? Is it likely? Explain your reasoning.



- **4.** With your classmates, conduct a simulation to represent polling a new sample of 1100 voters.
 - **a.** Divide 1100 by the number of students in your class to determine the size of each student's sample.

b. Generate an amount of random numbers equal to the sample size in part (a) to represent responses to the polling question. Generate random numbers between 1 and 100, with numbers from 1 to 54 representing support for re-electing the mayor and the numbers 55 to 100 representing support for not re-electing the mayor. Tally the results of your simulation, and then list the total number of tallies for each category.



Number of People Who Respond that They Will Vote to Re-elect the Mayor	Number of People Who Respond that They Will Not Vote to Re-elect the Mayor

c. Calculate the percent of people who state that they will vote to re-elect the mayor and the percent of people who state that they will vote to not re-elect the mayor based on your simulation.

d. Complete the simulation for the 1100 voters by combining the data from your classmates. List the percent of votes for each category.

Percent of People Who Respond that	Percent of People Who Respond that
They Will Vote to Re-elect the Mayor	They Will Vote to Not Re-elect the Mayor

e. Are the results of the simulation different from the results of the original poll? Explain.



f. If you conducted the simulation over and over, would you expect to get the same results or different results each time? Explain your reasoning.



The percent of voters who actually vote for the mayor in the election is the **population proportion**. The percent of voters in the sample who respond that they will vote for the mayor is the **sample proportion**. The population proportion and sample proportion are measures used for discrete, or categorical, data. For continuous data, these are called the population mean and sample mean.

When you and your classmates generated random numbers to simulate multiple samples of the 1100 voters, you came up with different sample proportions. The set of all of your classmates' sample proportions is part of a *sampling distribution*.

A **sampling distribution** is the set of sample proportions for all possible equal-sized samples. A sampling distribution will be close to a normal distribution, and the center of a sampling distribution is a good estimate of a population proportion—in this case, the percent of people who will actually vote to re-elect the mayor.

But rather than collecting a very large number of samples, a more practical method for estimating a population proportion is to use the sample proportion of a single sample to estimate the standard deviation of the sampling distribution. The standard deviation of a sampling distribution can give you a range in which the population proportion is likely to fall, relative to the sample proportion.

For example, to estimate the standard deviation of the sampling distribution for the sample of 1100 voters, you can use the formula $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$,

where (\hat{p}) is the sample proportion and *n* is the sample size.

R



You can learn the details of deriving the formula for the standard deviation of the sampling distribution, $\frac{\hat{p}(1-\hat{p})}{n}$, in a statistics course.

The sample proportion from the original poll is 54%, or 0.54. This is the percent of the 1100 people in the poll who said they would vote to re-elect the mayor.



\in	P	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.54(1-0.54)}{1100}}$
E	P	≈ 0.0150
\in	P	This means that 1 standard deviation below the sample proportion of 54% is
\in	B	54% - 1.5%, or $52.5%$. And 1 standard deviation above the sample proportion of $54%$ is $54% + 1.5%$, or $55.5%$.
\in	В	



5. Use the sample proportion and standard deviation of the sampling distribution to label the horizontal axis of the normal curve.



Percent Voting for Mayor's Re-election

6. Bobbie made an observation about the standard deviation of a sampling distribution.

Bobbie

The standard deviation of a sampling distribution gets smaller and smaller as the size of the sample gets larger and larger.

Is Bobbie's statement correct? Explain why or why not.



An estimated range of values that will likely include the population proportion or population mean is called a **confidence interval**. When stating the margin of error, a 95% confidence interval is typically used. However, other confidence intervals may also be used.

For example, the standard deviation of the sampling distribution for the election sample is 0.015, or 1.5%. Two standard deviations is 3%, so the margin of error is reported as $\pm 3\%$.

Confidence intervals for a population proportion are calculated using the sample proportion of a sample and the standard deviation of the sampling distribution.

- The lower bound of a 68% confidence interval ranges from 1 standard deviation below the sample proportion to 1 standard deviation above the sample proportion.
- The lower bound of a 95% confidence interval ranges from 2 standard deviations below the sample proportion to 2 standard deviations above the sample proportion.
- The lower bound of a 99.7% confidence interval ranges from 3 standard deviations below the sample proportion to 3 standard deviations above the sample proportion.



7. Determine each confidence interval for the election poll.

a. 68%

b. 95%

c. 99.7%

8. Explain the similarities and differences between each confidence interval for the election poll.



9. The result of the original poll was 54% with 3% margin of error. What confidence interval does 3% represent? Explain your reasoning.



- **10.** Use a 95% confidence interval to determine a margin of error and a range of values for each population proportion.
 - **a.** A survey of 1500 teenagers shows that 83% do not like waking up early in the morning.

2

b. A survey of 200 licensed high school students shows that 16% own their own car.



c. A survey of 500 high school students shows that 90% say math is their favorite class.

PROBLEM 2 Sweet Dreams: Exploring Continuous Data



A sample of 50 students at High Marks High School responded to a survey about their amount of sleep during an average night. The sample mean was 7.7 hours and the sample standard deviation was 0.8 hour.

Let's determine an estimate for the population mean sleep time for all High Marks High School students.

 If you gathered data from many new samples, would you expect the samples to have equal means or different means? Explain your reasoning.



Collecting additional samples of 50 students and plotting the sample mean of each sample will result in a sampling distribution. The sampling distribution will be approximately normal, and the mean of the sampling distribution is a good estimate of the population mean.

Just like with the categorical data, a more practical method for estimating the population mean amount of sleep for High Marks High School students is to use the sample mean to calculate an estimate for the standard deviation of the sampling distribution. The formula for the standard deviation of a sampling distribution for continuous data is $\frac{s}{\sqrt{n}}$, where s is the standard deviation of the original sample and *n* is the sample size.



2. Use the standard deviation from the original sample to determine the standard deviation for the sampling distribution. Explain your work.



3. Use the standard deviation of the sampling distribution to determine a 95% confidence interval for the population mean. Explain your work.

4. Write the 95% confidence interval in terms of the population mean plus or minus a margin of error.

- **5.** Use a 95% confidence interval to determine a range of values for each population mean.
 - **a.** A sample of 75 students responded to a survey about the amount of time spent online each day. The sample mean was 3.2 hours, and the standard deviation of the sampling distribution was 0.9 hour.

b. A sample of 1000 teachers responded to a survey about the amount of time they spend preparing for class outside of school hours. The sample mean was 2.5 hours, and the standard deviation of the sampling distribution was 0.5 hour.



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c. A sample of 400 adults responded to a survey about the distance from their home to work. The sample mean was 7.8 miles, and the standard deviation of the sampling distribution was 1.6 miles.

Talk the Talk



1. What is the difference between a sample and a sampling distribution?

2. What is the difference between a sample proportion and a sample mean?

Online Time Study, Part III

To summarize data from a sample survey, observational study, or experiment:

- Calculate measures of center.
- Calculate measures of spread.
- Select the most appropriate method(s) to display the data (dot plot, histogram, stem-and-leaf plot, box-and-whisker plot, normal curve).
- Describe the characteristics of the graphical display.

To analyze data from a sample survey, observational study, or experiment:

 Use confidence intervals to determine a range of values for the population mean(s) or proportion(s).



Recall the study described in previous lessons about the amount of time students in your school are online each day.

1. Will your study involve estimating a population mean or a population proportion? Explain your reasoning.

2. Use a 95% confidence interval to determine a range of values for the population mean, given a random sample of 60 students with a sample mean of 3.5 hours and a standard deviation of 1.1.

3. Use the sample mean and standard deviation of the sampling distribution to label the horizontal axis of the normal curve.



Average Time Spent Online



Be prepared to share your results and methods.

2.4

How Much Different?

Using Statistical Significance to Make Inferences About Populations

LEARNING GOALS

In this lesson, you will:

- Use sample proportions to determine whether differences in population proportions are statistically significant.
- Use sample means to determine whether differences in population means are statistically significant.

KEY TERM

statistically significant

A person's blood pressure is typically measured using two numbers. One number represents the pressure in the arteries when the heart beats. This is the systolic pressure. The other number represents the pressure in the arteries between heartbeats. This is the diastolic pressure. For example, $\frac{118}{74}$ represents a systolic pressure of 118 and a diastolic pressure of 74.

Commercials on a local TV station claim that Whatta Water tastes better than tap water, but a local news anchor does not believe the claim. She sets up an experiment at a local grocery store to test the claim. A representative, unbiased sample of 120 shoppers participate in the tasting survey using unmarked cups. Out of the 120 people, 64 said Whatta Water tastes better than tap water.



2

1. If shoppers had to choose one or the other and there was no difference in the tastes of the two waters, what proportion of shoppers would you expect to say that Whatta Water tastes better? Explain your reasoning.

2. What is the sample proportion of shoppers who stated that Whatta Water tastes better?



3. Based on your answers to Questions 1 and 2, what reason(s) can you give to doubt Whatta Water's claim? Explain your reasoning.



The term **statistically significant** is used to indicate that a result is very unlikely to have occurred by chance. Typically, a result that is more than 2 standard deviations from the mean, or outside a 95% confidence interval, is considered statistically significant.

 Use a 95% confidence interval to determine a range of values for the population proportion of people who prefer the taste of Whatta Water. Explain your work. two standard deviations from the mean seems to come up a lot!

5. Use the sample proportion and standard deviation of the sampling distribution to label the horizontal axis of the normal curve.



Percent Who Prefer Whatta Water to Tap Water



6. Based on the range of values of the 95% confidence interval, what conclusion can you make about Whatta Water's claim that their water tastes better than tap water?



- 7. The local water company also conducted a survey of 120 people which they said showed that people prefer tap water over Whatta Water. Forty-one of the respondents said Whatta Water tastes better.
 - **a.** Use a 95% confidence interval to determine a range of values for the population proportion of people who prefer Whatta Water. Explain your work.



b. Use the sample proportion and standard deviation of the sampling distribution to label the horizontal axis of the normal curve.



Percent Preferring Whatta Water

c. Based on the range of values of the 95% confidence interval, what conclusion can you draw about the local water company's claim that tap water tastes better than Whatta Water?

8. Use a random number generator to conduct a simulation of the local water company's survey, for a new sample of 120 people. Generate a random number between 1 and 100, with numbers from 1 to 34 representing that Whatta Water tastes better and numbers from 35 to 100 representing that tap water tastes better. List the results in the table.

Percent of People in Simulation Who Said Whatta Water Tastes Better	Percent of People in Simulation Who Said Tap Water Tastes Better

9. On the normal curve in Question 7 part (b), locate and mark the sample proportion of your simulation. Describe the location of the sample proportion on the normal curve.



10. Compare the results of your simulation with the water company's study and with Whatta Water's study. Are your results significantly different? Explain your reasoning.

PROBLEM 2 Nonstop Homework: Exploring Continuous Data



A sample of 40 students at High Marks High School responded to a survey about the average amount of time spent on homework each day. The sample mean was 2.9 hours and the sample standard deviation was 0.8 hour.

- 1. Use a 95% confidence interval to determine a range of values for the population mean. Explain your work.



2. Label the horizontal axis of the normal curve that represents the sampling distribution.



Mean Hours Spent on Homework

- 3. A new sample of 40 students was taken and the resulting sample mean was 2.70 hours.
 - **a.** On the normal curve in Question 2, locate and mark the sample mean of the new sample. Describe the location of the sample mean on the normal curve.

b. Are the results of the new sample statistically significant? Explain your reasoning.



4. What sample mean values are statistically significant? Explain your reasoning.



5. Mary shared a comment about the time she spends on homework.

Mary

I spend an average of 3.5 hours on homework every night. Compared to the sample mean, the average amount of time I spend on homework every night is statistically significant.

Is Mary's reasoning valid? Explain why or why not.



PROBLEM 3 Read Between the Lines: Comparing Categorical Data



Two hometown newspapers conducted a poll about whether residents are for or against a tax to provide funding for school renovations in the district. Today's News polled 75 residents and 53 stated that they are in favor of the tax increase. Local Time polled 100 residents and 54 stated they are in favor of the tax increase.



Ζ

1. Calculate the sample proportion for each poll.

2. Use the results from each poll to estimate a range of values for the population proportion using a 95% confidence interval. Explain your work.

3. The Reporter newspaper published a survey of 90 residents and 38 stated that they are in favor of the tax increase. Use a 95% confidence interval to determine a range of values for the population proportion. Explain your work.

2

If two confidence intervals overlap, then the difference between the population proportions or population means is not statistically significant. If the intervals do not overlap then the difference between the population proportions or population means is statistically significant.

- **4.** Compare the population proportion estimates and determine whether their differences are statistically significant. Explain your reasoning.
 - a. The Reporter and Local Times



b. The Reporter and Today's News

PROBLEM 4 Pressure Situation: Comparing Continuous Data

A researcher conducted a randomized experiment to see whether there was a link between a new supplement and blood pressure. She collected data from a representative, unbiased sample of 200 people who had high blood pressure. One hundred of the people were randomly selected to take the supplement and the other 100 people were given a placebo. Recall that a placebo is a treatment that is assumed to have no real effect on the characteristic of interest.

The participants' blood pressures were recorded at the beginning and at the end of the 12-week experiment, and the difference (end – beginning) was calculated.

 For the 100-person treatment that took the placebo, what value would you expect for the difference of sample means at the beginning of the experiment and at the end of the experiment. Explain your reasoning.



2. For the 100-person treatment that took the supplement, what value would you expect for the difference of sample means at the beginning of the experiment and at the end of the experiment. Explain your reasoning.



Suppose that the mean difference in blood pressure of the group who took the supplement was -15 with a standard deviation of 3.2, and the mean difference in blood pressure of the group who took the placebo was 1.7 with a standard deviation of 0.3.

3. Interpret and explain the meaning of a negative mean difference for the treatment that took the supplement and a positive mean difference for the treatment that took the placebo.

4. Use a 95% confidence interval to determine a range of values for the population mean of each treatment. Explain your work.



5. What conclusion can you make about whether or not the supplement effectively lowers high blood pressure? Explain your reasoning.



PROBLEM 5 Decisions, Decisions . . .

A manufacturing company has a policy that states that if significantly more than 2% of computer parts are defective during an 8-hour shift, then the parts from that shift will not be shipped. During an 8 hour shift, 1020 parts were produced and 22 were defective. Should the parts be shipped? Explain your reasoning.



2. The mean grade point average (GPA) of a random sample of 50 High Mark High School students who had a part-time job during the previous grading period is 3.15 with a standard deviation of 0.44. The mean GPA of a random sample of 50 High Mark High School students who did not have a part-time job during the previous grading period is 2.77 with a standard deviation of 0.35. Does that data suggest a possible link between High Mark High School students' part-time job status and their GPA?

PROBLEM 6 The End of the Line



Recall the problem from the previous lesson about part-time job status and grade point average (GPA).

The population mean interval for the GPA of High Mark High School students who have a part-time job, 3.03 to 3.27, does not overlap with population mean interval for the GPA of High Mark High School students who do not have a part-time job, 2.67 to 2.87.



Carmen share	ed a conclusion about part-time job status and GPA.
	Carmen
	Because the results of the statistical analysis
	are statistically significant, I can conclude
	that holding a part-time job will result in a
	higher GPL.
Is Carmen's s	statement correct? Explain why or why not.

The interval for the estimate of the population mean for the GPA of neighboring Great Beginnings High School students who do not have a part-time job is 3.18 to 3.39.

2. Is the GPA of students who do not have a part-time job statistically different at High Mark High and Great Beginnings High School? Explain your reasoning.

3. The estimate for the population mean for the math GPA of Great Beginnings High School students using a sample of the math club is 3.27 to 3.54. The estimate for the population mean for the math GPA of Great Beginnings High School students using a sample of the government club is 3.11 to 3.40.

Мах

The results of the statistical analysis are not statistically significant because the population mean intervals for math GPA overlap.



Is Max's statement correct? Explain why or why not.

Online Time Study, Part IV



To analyze data from a sample survey, observational study, or experiment, you can use statistical significance to make inferences about populations.

Recall the study you have been planning about the amount of time students in your school are online each day.



Suppose two samples of data were collected. One sample of 40 students in your school has a sample mean of 2.3 hours and a standard deviation of 0.7 hour. Another sample of 40 students in your school has a sample mean of 3.7 hours and a standard deviation of 1.1 hours.

 Use a 95% confidence interval to determine whether the estimate of the population means using each sample is statistically significant. Explain your work.





Be prepared to share your results and methods.

2.5

DIY Designing a Study and Analyzing the Results

LEARNING GOALS

In this lesson, you will:

- Analyze the validity of conclusions based on statistical analysis of data.
- Design a sample survey, observational study, or experiment to answer a question.
- Conduct a sample survey, observational study, or experiment to collect data.
- Summarize the data of your sample survey, observational study, or experiment.
- Analyze the data of your sample survey, observational study, or experiment.
- Summarize the results and justify conclusions of your sample survey, observational study, or experiment.

DIY stands for "do it yourself." So, why not? Try to write an interesting opener yourself for this lesson. Use these hints to help you get started:

- Make your opener related to something about the lesson or the whole chapter.
- Write about something you think other students would be interested in reading.
- Be creative!

Share your opener with your classmates. Which one did you like best?

PROBLEM 1 Do It Yourself!



Use the following guidelines to design and conduct a sample survey, observational study, or experiment, summarize and analyze the data, and draw conclusions. You can use this page as a checklist while planning and conducting your study.

I.	Design a sample survey, observational study, or experiment.	
	 Select a characteristic of interest to learn about from a sample survey, observational study, or experiment. 	
	• Select a question that can be answered by collecting quantitative data.	
	 Identify the population. 	
	Identify the characteristic being studied.	
	Describe the method for choosing a random sample.	
	Address potential sources of bias.	
١١.	Conduct the sample survey, observational study, or experiment.	
	 Use the sampling method to collect data for your sample survey, observational study, or experiment. 	
III.	Summarize the data of the sample survey, observational study, or experiment.	
	Calculate measures of center.	
	Calculate measures of spread.	
	 Select the most appropriate method(s) to display the data (dot plot, histogram, stem-and-leaf plot, box-and-whisker plot, normal curve). 	
	 Describe the characteristics of the graphical display. 	
IV.	Analyze the data of the sample survey, observational study, or experiment.	
	• Use confidence intervals to determine a range of values for the population mean(s) or proportion(s).	
	Using statistical significance to make inferences about populations.	
V.	Draw conclusions based on the results of the sample survey, observational study, or experiment.	
	• Write a conclusion that answers the question of interest of your sample survey, observational study, or experiment. Use the data and data analysis to justify your conclusion.	

2



Be prepared to share your results and methods.

Chapter 2 Summary

KEY TERMS

- characteristic of interest (2.1)
- sample survey (2.1)
- random sample (2.1)
- biased sample (2.1)
- observational study (2.1)
- experiment (2.1)
- treatment (2.1)
- experimental unit (2.1)confounding (2.1)

- convenience sample (2.2)
- subjective sample (2.2)
- volunteer sample (2.2)simple random
- sample (2.2)stratified random
 - sample (2.2)
- cluster sample (2.2)
 - cluster (2.2)

- systematic sample (2.2)
- parameter (2.2)
- statistic (2.2)
- population proportion (2.3)

2

- sample proportion (2.3)
- sampling distribution (2.3)
- confidence interval (2.3)
- statistically significant (2.4)

2.1 Identifying Characteristics of Sample Surveys, Observational Studies, and Experiments

The characteristic of interest is the specific question to be answered or the specific information to be gathered for sample surveys, observational studies, and experiments. The entire set of items from which data can be selected is the population. A subset of the population that is selected is a sample.

Example

Fifty-five deer are randomly selected from a park in the township. They are anesthetized, weighed, and then released back into the park.

The population is all of the deer in the park. The sample is the 55 deer selected. The characteristic of interest is the mean weight of the deer.

Differentiating Between Sample Surveys, Observational Studies, and Experiments

A sample survey poses a question of interest to a sample of the targeted population. An observational study gathers data about a characteristic of the population without trying to influence the data. An experiment gathers data on the effect of one or more treatments on the characteristic of interest.

Example

A study states that approximately 78% of planes arrived on time during a 3 hour period at an airport.

This is an observational study since the study only gathered data about the number of planes that arrived on time and did not try to influence the data.

2.2

2.1

Using a Variety of Sampling Methods to Collect Data

Sampling methods could include convenience sampling, volunteer sampling, simple random sampling, stratified random sampling, cluster sampling, and systematic sampling.

Example

The data set below shows the number of late student arrivals at four elementary schools each week for five weeks.

Number of Late Arrivals										
Week 1	Week 2	Week 3	Week 4	Week 5						
49	37	45	44	43						
47	41	45	46	48						
39	43	38	44	42						
43	47	39	39	42						
52	55	50	54	55						

You can create a stratified random sample with 5 data values to describe the number of late arrivals by randomly choosing one school from each of the 5 weeks and recording the number of late arrivals: {39, 37, 50, 46, 42}.

2.2 Identifying Factors of Sampling Methods that could Contribute to Gathering Biased Data

Some sampling methods introduces bias, which reduces the likelihood of a representative, unbiased sample.

Example

A cereal company conducts taste tests for a new cereal on a random sample of its employees.

There is bias in this study because the taste test is only conducted on the company's employees. It is possible that the employees will prefer the cereal of the company that employs them for other reasons than taste.



Exploring, Identifying, and Interpreting the Role of Randomization in Sampling

You can use random sampling by using a random digit table or a graphing calculator to create unbiased samples.

Example

For the data set, you can use a calculator to generate four random numbers between 1 and 10. Then you can use the numbers generated to create a random sample of four from the data set.

The 25-meter freestyle times, in seconds, of ten young swimmers are shown.

Swimmer	1	2	3	4	5	6	7	8	9	10
Time	21.2	19.3	18.7	20.6	20.5	18.4	22.9	23.5	18.2	17.9

Possible random numbers: 19.3, 18.7, 22.9, 17.9.

Recognizing that Data from Samples are Used to Estimate Population Proportions and Population Means

Data from samples are used to calculate confidence intervals that estimate population proportions and population means.

Example

A sample of 250 women responded to a survey about the amount of money they spend on cosmetics each month. The sample mean was \$45.50 and the sample standard deviation was \$10.75.

The interval from \$44.14 to \$46.86 represents a 95% confidence interval for the population mean.

 $\frac{s}{\sqrt{n}} = \frac{10.75}{\sqrt{250}} \approx 0.68$

2.4

2.3

Using Sample Proportions to Determine Whether Differences in Population Proportions are Statistically Significant

To determine the sample proportions that would be statistically significant, use the normal curve and label it based on the standard deviation from the sample.

Example

Use the sample proportion and standard deviation of the sampling distribution to label the horizontal axis of the normal curve. Then, determine what sample proportions would be statistically significant.

A sample proportion of families that own dogs is 74%, and the standard deviation is 0.017.



Sample proportion values less than 70.6% and greater than 77.4% are statistically significant because those values are outside of the 95% confidence interval.

2.4 Using Sample Means to Determine Whether Differences in Population Means are Statistically Significant

Use the sample mean and standard deviation to determine the margin of error for the confidence interval. The margin of error is 2 times the standard deviation.

Example

Use a 95% confidence interval to determine a range of values for the population mean. Explain your work.

A sample of 80 doctors took a stress test. The sample mean was 44.5 and the sample standard deviation was 14.8.

The interval from 41.2 to 44.5 represents a 95% confidence interval for the population mean.

The margin of error is approximately ± 3.30 .

$$\frac{s}{\sqrt{n}} = \frac{14.8}{\sqrt{80}} \approx 1.65$$

2(1.65) = 3.30

2.5 Conducting a Sample Survey, Observational Study, or Experiment to Answer a Question

You can determine what type of sample technique would be most appropriate to answer a question for a sample survey, observational study, or experiment.

Example

Suppose you want to estimate the number of senior citizens in a town that are on public assistance.

You can assign all the senior citizens in the town an ID number and use a computer to randomly generate a sample of senior citizens. This technique provides a random sample of the population of the senior citizens in the town, and random sampling is typically representative of a population.