

Questions on 4.5?

Practice below:

Determine what the end behavior is approaching for the following rational expressions.

$$\frac{d0}{d1} \quad \frac{40}{24r - 16}$$

$$EB \rightarrow 0$$

$$\frac{n^2 + 3n - 10}{n + 5} \quad \frac{d2}{d1}$$

$$E.B \rightarrow \pm \infty$$

4.5 Watch Your "Behavior"

A Develop Understanding Task

In this task, you will develop your understanding of the end behavior of rational functions as well as discover the behavior of even and odd functions.

Part I: End behavior of rational functions

After completing the task *The Gift*, Marcus and Hannah were talking about the discussion regarding the end behavior of the parent function $f(x) = \frac{1}{x}$. Marcus said "I thought the end behavior of all functions was that you either ended up going to positive or negative infinity." Hannah agreed, adding "Now we have a function that approaches zero. I wonder if all rational functions will always approach zero as x approaches $\pm\infty$." Marcus replied "I am sure they do. Just like all polynomial functions end behavior approaches either $\pm\infty$, I think the end behavior for all rational functions must approach zero".

1. Could Marcus be right? Make a conjecture about the end behavior of rational functions and test it. (Hint: this should take awhile- be sure to think about the various rational expressions we have studied). As you analyze the end behavior of different rational functions, try to generalize the patterns you notice regarding end behavior.

Mason's side
proper
 $n < d$

end behavior $\rightarrow 0$
bc horizontal asy. @ $y=0$
(x-axis)

$$\frac{(x^2 - 1)}{(2x^3 + 4x)}$$

$$\frac{-5x^4 - 2x^3}{7x^6}$$

E.B. $\rightarrow 0$
horizontal asy. @ $y=0$

Sawyer's side
improper
 $n > d$

end beh $\rightarrow \pm\infty$
bc slant asy. is quotient when we long divide

$$\frac{7x^4 + 2x}{3x^2 - 7}$$

$$\frac{-3x^7 + 2x}{8x^4 - 5}$$

E.B. $\rightarrow \pm\infty$

Dar's side
improper
 $n = d$

end beh \rightarrow ratio of leading coefficients

$$\frac{8x^5}{4x^5 + 1}$$

$$\frac{-3x^7 + 2}{1x^7 - 7}$$

$\rightarrow 2$
 $\rightarrow -3$

• end beh \rightarrow ratio of leading coefficients
• horizontal asy. at ratio of leading coeff.

HW: # 1-12

4.3 #21

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$$\frac{x^3 - 5x + 2}{x - 10} = 4$$

$$x^3 - 9x + 42 = 0$$

$$x = -4.33$$

Part II: Even and Odd Functions

Below are three graphs: one represents an even function, one represents an odd function, and one is neither even nor odd.

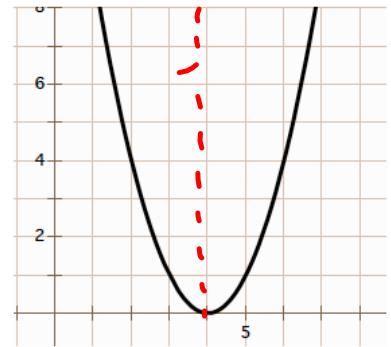
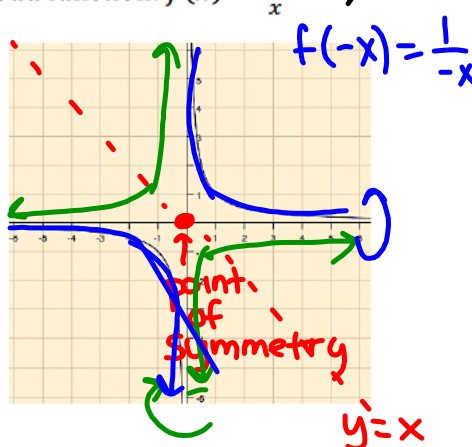
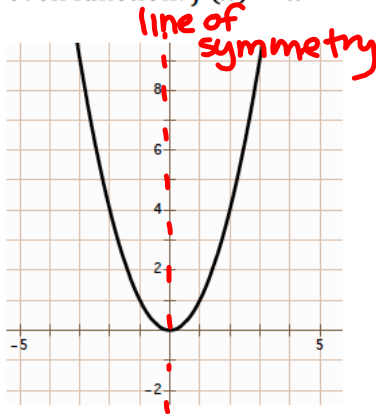
$$3^{-1} = \frac{1}{3^1}$$

$$x^2 - 8x + 16$$

even function: $f(x) = x^2$

odd function: $f(x) = \frac{1}{x} = x^{-1}$

neither: $f(x) = (x - 4)^2$



2. Use the graphs and their corresponding functions to write a definition for an even function and an odd function.

- A function is an even function if... there is symmetry about the y-axis and if $f(-x) = f(x)$.

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = x^2 \quad f(-x) = f(x)$$

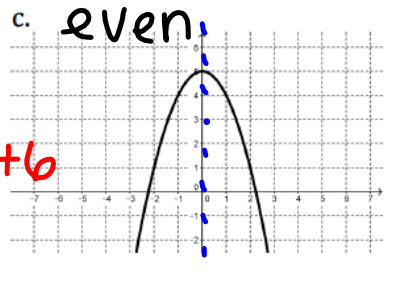
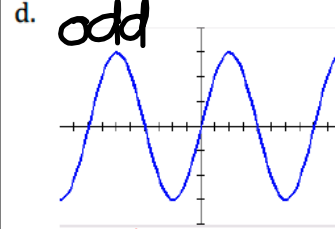
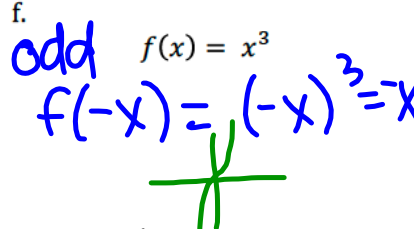
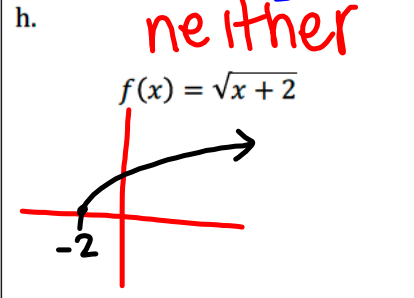
- A function is an odd function if... there is symmetry about the origin and if $f(-x) = -f(x)$

$$f(x) = x^3 + 2x$$

$$f(-x) = (-x)^3 + 2(-x) = -x^3 - 2x$$

$$f(-x) = -f(x)$$

3. Below are more functions. Based on your definition, classify as either even, odd, or neither.

<p>a. even</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>10</td> </tr> <tr> <td>-1</td> <td>5</td> </tr> <tr> <td>0</td> <td>-4</td> </tr> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>10</td> </tr> </tbody> </table> <p>$f(-x)$</p> <p>$f(x)$</p>	x	y	-2	10	-1	5	0	-4	1	5	2	10	<p>b.</p> $f(x) = x^4 - 3x + 6$ <p>neither</p> $f(-x) = (-x)^4 - 3(-x) + 6$ $f(-x) = x^4 + 3x + 6$	<p>c. even</p> 
x	y													
-2	10													
-1	5													
0	-4													
1	5													
2	10													
<p>d. odd</p> 	<p>e.</p> $f(x) = x^4 - 3$ <p>even</p> $f(-x) = (-x)^4 - 3$ $f(-x) = x^4 - 3$	<p>f.</p> <p>odd</p> $f(x) = x^3$ $f(-x) = (-x)^3 = -x^3$ 												
<p>g. odd</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>-10</td> </tr> <tr> <td>-1</td> <td>-5</td> </tr> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>10</td> </tr> </tbody> </table> <p>$f(-x)$</p> <p>$f(x)$</p>	x	y	-2	-10	-1	-5	0	0	1	5	2	10	<p>h. neither</p> $f(x) = \sqrt{x+2}$ 	<p>i. odd</p> $f(x) = x(x-2)(x+2)$ $f(-x) = -x(-x-2)(-x+2)$
x	y													
-2	-10													
-1	-5													
0	0													
1	5													
2	10													

4. The answers to question three are at the bottom of this page. Check your solutions and adjust your definitions of even and odd functions, as needed.

Simplify.

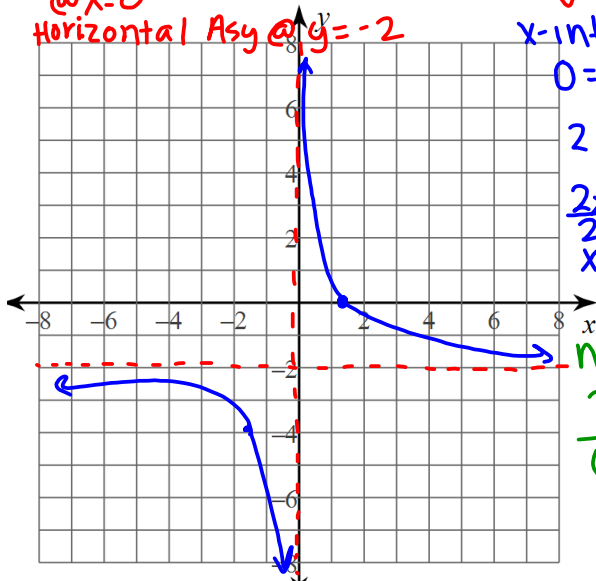
$$\frac{2n^2 + 2n}{(n+1)} = \frac{2n(n+1)}{(n+1)} = 2n$$

$$\frac{k^2 - 17k + 72}{k^2 - k - 56} = \frac{(k-8)(k-9)}{(k-8)(k+7)} = \frac{k-9}{k+7}$$

Graph. $f(x) = \frac{3}{x} - 2$

Vert. Asy @ $x=0$

Horizontal Asy @ $y=-2$



parent rational function $f(x) = \frac{1}{x}$

$\downarrow 2$

x-int:
 $0 = \frac{3}{x} - 2$
 $2 = \frac{3}{x}$
 $\frac{2x}{2} = \frac{3}{2}$
 $x = 1.5$

no y-int. $\frac{3}{0} - 2$ is undefined

Vert. Asy $x = -1$

Horizontal Asy $y = -3$

x-int
 $\frac{3}{3} = -\frac{2}{x+1}$

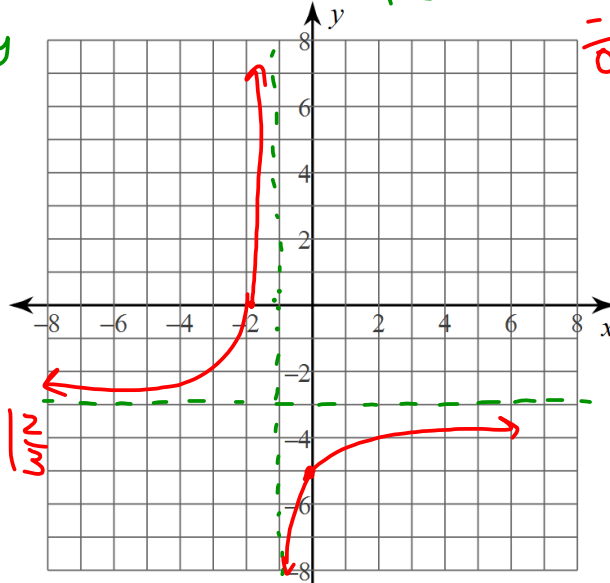
$3x + 3 = -2$

$\frac{3x}{3} = \frac{-5}{3}$

$x = -\frac{5}{3} = -1\frac{2}{3}$

$f(x) = -\frac{2}{x+1} - 3$
 reflected $x+1$ \leftarrow $\downarrow 3$ dilated

y-int
 $\frac{-2}{0+1} - 3$
 $-2 - 3$
 -5



Homework/Classwork

Finish the "Ready, Set, Go"
problems from lesson 4.5 & the
"Rational Function
Supplementary Assignment"
WKS