

**Secondary Math III Honors  
Module 3 Study Guide  
Polynomials**

**Directions: Show all work.**

**Identify the following functions as linear, exponential, quadratic, cubic, or logarithmic.**

<p>1. <i>expon.</i></p> <table border="1" style="display: inline-table; margin-right: 10px;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>8</td></tr> </table>	x	y	0	1	1	2	2	4	3	8	<p>2. <i>cubic...</i></p> <table border="1" style="display: inline-table; margin-right: 10px;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>3</td><td>27</td></tr> </table>	x	y	0	0	1	1	2	8	3	27	<p>3. <i>logarithm (linear)</i></p> <table border="1" style="display: inline-table; margin-right: 10px;"> <tr><th>x</th><th>y</th></tr> <tr><td>1</td><td>0</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>4</td><td>2</td></tr> <tr><td>8</td><td>3</td></tr> </table>	x	y	1	0	2	1	4	2	8	3	<p>4. <i>Quadratic</i></p> <table border="1" style="display: inline-table; margin-right: 10px;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>7</td></tr> </table>	x	y	0	1	1	2	2	4	3	7	<p>5. <i>linear</i></p> <table border="1" style="display: inline-table;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>3</td></tr> <tr><td>3</td><td>4</td></tr> </table>	x	y	0	1	1	2	2	3	3	4
x	y																																																					
0	1																																																					
1	2																																																					
2	4																																																					
3	8																																																					
x	y																																																					
0	0																																																					
1	1																																																					
2	8																																																					
3	27																																																					
x	y																																																					
1	0																																																					
2	1																																																					
4	2																																																					
8	3																																																					
x	y																																																					
0	1																																																					
1	2																																																					
2	4																																																					
3	7																																																					
x	y																																																					
0	1																																																					
1	2																																																					
2	3																																																					
3	4																																																					

**Use the equations below to answer questions 6-9.**

$f(x) = x + 1$                        $g(x) = x^2 + 2$                        $h(x) = x^3 + 3$

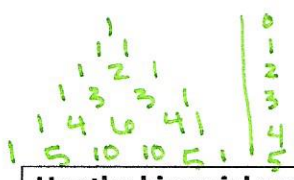
<p>6. Find <math>f(x) + g(x)</math></p> <p style="text-align: center;"><i><math>x + 1 + x^2 + 2</math></i></p> <p style="text-align: center;"><i><math>x^2 + x + 3</math></i></p>	<p>7. Find <math>f(x) - g(x)</math></p> <p style="text-align: center;"><i><math>(x + 1) - (x^2 + 2)</math></i></p> <p style="text-align: center;"><i><math>x + 1 - x^2 - 2</math></i></p> <p style="text-align: center;"><i><math>x^2 + x - 1</math></i></p>
---	--

<p>8. Find <math>f(x) \cdot g(x)</math></p> <p style="text-align: center;"><i><math>(x + 1)(x^2 + 2)</math></i></p> <p style="text-align: center;"><i><math>x^3 + 2x + x^2 + 2</math></i></p> <p style="text-align: center;"><i><math>x^3 + x^2 + 2x + 2</math></i></p>	<p>9. Find <math>f(x)[h(x) + g(x)]</math></p> <p style="text-align: center;"><i><math>(x + 1)[x^2 + 2 + x^3 + 3]</math></i></p> <p style="text-align: center;"><i><math>(x + 1)(x^3 + x^2 + 5)</math></i></p> <p style="text-align: center;"><i><math>x^4 + x^3 + 5x + x^3 + x^2 + 5</math></i></p> <p style="text-align: center;"><i><math>x^4 + 2x^3 + x^2 + 5x + 5</math></i></p>
---	--

**Solve for x for questions 10-12.**

<p>10. <math>9x^2 - 25 = 0</math></p> <p style="text-align: center;"><i><math>(3x + 5)(3x - 5)</math></i></p> <p style="text-align: center;"><i><math>x = \frac{5}{3}, -\frac{5}{3}</math></i></p>	<p>11. <math>x^2 + 4x = -3</math></p> <p style="text-align: center;"><i><math>x^2 + 4x + 3 = 0</math></i></p> <p style="text-align: center;"><i><math>(x + 3)(x + 1)</math></i></p> <p style="text-align: center;"><i><math>x = -3, -1</math></i></p>	<p>12. <math>(x + 4)(x - 3)(x + 1) = 0</math></p> <p style="text-align: center;"><i><math>x = -4, +3, -1</math></i></p>
--	---	---

<p>13. Write a polynomial in <b>factored form</b> that has a leading coefficient of 2, and the following roots: -1, 2, -3.</p> <p style="text-align: center;"><i><math>2(x + 1)(x - 2)(x + 3)</math></i></p>	<p>14. Write a polynomial in <b>standard form</b> that has a leading coefficient of 2, and the following roots: -1, 2, -3.</p> <p style="text-align: center;"><i><math>2(x + 1)(x - 2)(x + 3)</math></i></p> <p style="text-align: center;"><i><math>(2x + 2)(x^2 + x - 6)</math></i></p> <p style="text-align: center;"><i><math>2x^3 + 2x^2 - 12x + 2x^2 + 2x - 12</math></i></p> <p style="text-align: center;"><i><math>2x^3 + 4x^2 - 10x - 12</math></i></p>
--	---



$$(x+y)^n = x^n y^0 + x^{n-1} y^1 + \dots$$

Use the binomial expansion and Pascal's triangle to solve questions 15-16.

15. What is the third term in the expansion of  $(x+7)^4$ ?

$x=x$   
 $y=7$   
 $n=4$

$$6x^2y^2$$

$$6x^2(7^2)$$

$$294x^2$$

16. What is the second term in the expansion of  $(2+y)^3$ ?

$x=2$   
 $y=y$   
 $n=3$

$$3x^2y^1$$

$$3x^2$$

17. Write a polynomial in factored form and standard form with a leading coefficient of -3, and the following roots: 4, 2i, and -2i.

$$-3(x-4)(x-2i)(x+2i)$$

$$(-3x+12)(x^2+4)$$

$$-3x^3 - 12x + 12x^2 + 48$$

$$-3x^3 + 12x^2 - 12x + 48$$

For 18-19: Use the Remainder Theorem to determine if the following are roots of the given polynomial or not; state the remainder.

18.  $(x^3 + 3x^2 - 59x + 30) \div (x-6)$

$$6^3 + 3 \cdot 6^2 - 59 \cdot 6 + 30 = 0$$

factor; R=0

19.  $(n^3 + n^2 - 28n + 28); f(-4)$

$$(-4)^3 + (-4)^2 - 28(-4) + 28 = 92$$

not factor; R=92

For 20-21: Divide the following polynomials.

20.  $(3n^2 - 16n^2 + 20n - 4) \div (3n - 1)$

$$\begin{array}{r} n^2 + 5n + 5 \\ 3n-1 \overline{) 3n^2 - 16n^2 + 20n - 4} \\ \underline{-(3n^3 - n^2)} \phantom{-4} \\ -15n^2 + 20n \phantom{-4} \\ \underline{-(-15n^2 + 5n)} \phantom{-4} \\ \phantom{-15n^2} 15n - 4 \\ \phantom{-15n^2} \underline{-(15n - 5)} \\ \phantom{-15n^2} \phantom{15n} 1 \end{array}$$

$n^2 - 5n + 5 + \frac{1}{3n-1}$

21.  $(p^3 + 3p^2 + 2) \div (p + 3)$

$$\begin{array}{r} p^2 \\ p+3 \overline{) p^3 + 3p^2 + 0p + 2} \\ \underline{-(p^3 + 3p^2)} \phantom{+2} \\ 0 + 0 + 2 \end{array}$$

$p^2 + \frac{2}{p+3}$

Using the rational root theorem, factor the following polynomials completely and state both the real and complex roots.

22.  $x^3 + 9x^2 + 15x + 7 = 0$

$p: \pm \{7, 1\}$   
 $q: 1$

$f(1) = 896$   
 $f(-1) = 0$   
 $f(1) = 32$

$$\begin{array}{r} x^2 + 8x + 7 \rightarrow (x+1)(x^2 + 8x + 7) \\ x+1 \overline{) x^3 + 9x^2 + 15x + 7} \\ \underline{-(x^3 + x^2)} \phantom{+7} \\ 8x^2 + 15x \phantom{+7} \\ \underline{-(8x^2 + 8x)} \phantom{+7} \\ \phantom{8x^2} 7x + 7 \\ \phantom{8x^2} \underline{-(7x + 7)} \\ \phantom{8x^2} \phantom{7x} 0 \end{array}$$

$(x+1)(x+1)(x+7)$   
 $(x+1)^2(x+7)$   
Real roots: -1 (mult: 2), -7

NO complex roots

23.  $x^4 + 3x^2 - 40 = 0$

$p: 40 \rightarrow 1, 2, 4, 5, 8, 10, 20, 40$   
 $q: 1 \rightarrow 1$

$f(1) = -36$   
 $f(-1) = -36$   
 $f(2) = -12$   
 $f(-2) = -12$   
 $f(4) = 264$   
 $f(-4) = 264$   
 $f(5) = 600$   
 $f(-5) = 600$   
 $f(8) = 4248$   
 $f(-8) = 4248$

$$(x^2 + 8)(x^2 - 5)$$

$$(x + 2i\sqrt{2})(x - 2i\sqrt{2})(x + \sqrt{5})(x - \sqrt{5})$$

Real roots:  $\sqrt{5}, -\sqrt{5}$

Complex roots:  $2i\sqrt{2}, -2i\sqrt{2}$

Graph the following functions, make sure to label all points clearly.

22.  $f(x) = (x + 2)^2(x - 3)^2$

4.9

Degree of function: 4

Even or odd degree: even

Positive or negative leading coefficient:

pos

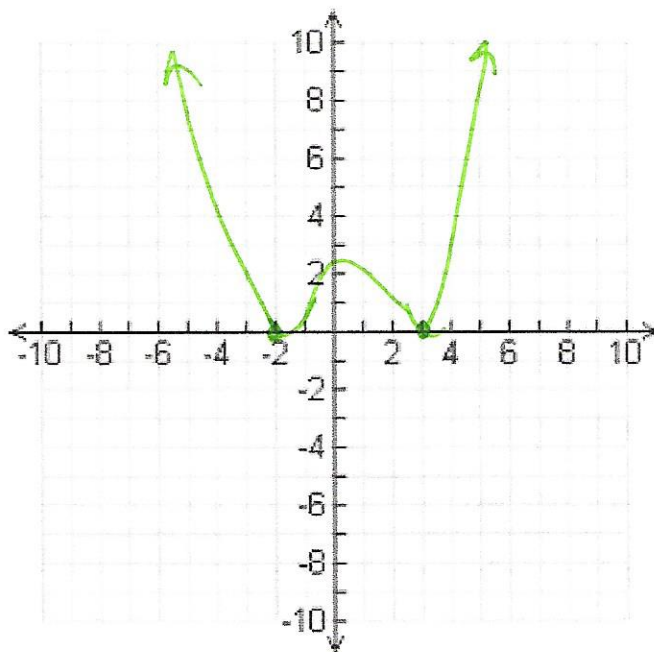
End Behavior:

As  $x \rightarrow -\infty, f(x) \rightarrow \infty$ .

As  $x \rightarrow \infty, f(x) \rightarrow \infty$ .

Roots, including multiplicity:

-2 m:2 3 m:2



23.  $f(x) = -(x - 4)^4(x - 1)^2$

Degree of function: 6

Even or odd degree: even

Positive or negative leading coefficient:

neg

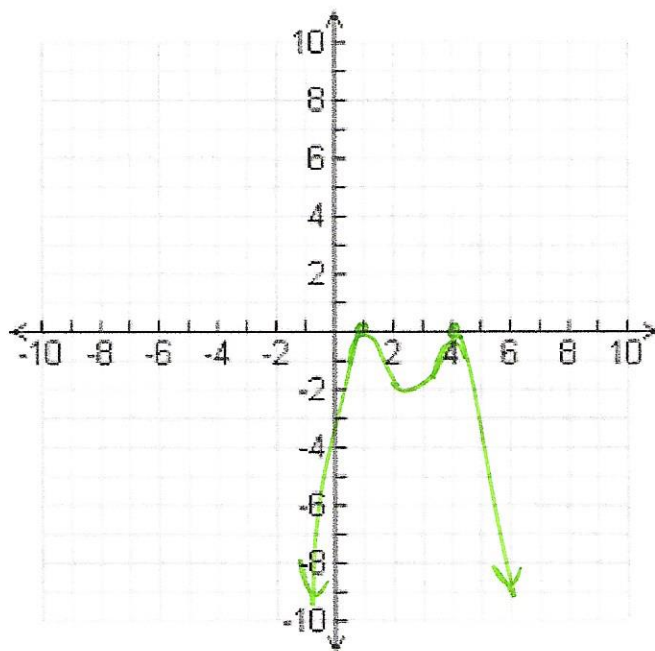
End Behavior:

As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$ .

As  $x \rightarrow \infty, f(x) \rightarrow -\infty$ .

Roots, including multiplicity:

4 m:4 ; 1 m:2



24.  $f(x) = x(x^2 + 4)$

Degree of function: 3

Even or odd degree: odd

Positive or negative leading coefficient:

pos

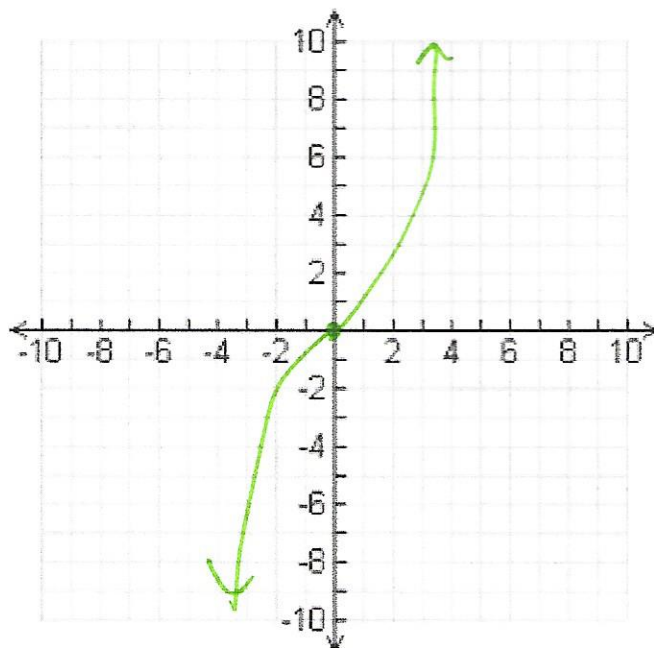
End Behavior:

As  $x \rightarrow -\infty, f(x) \rightarrow -\infty$ .

As  $x \rightarrow \infty, f(x) \rightarrow \infty$ .

Roots, including multiplicity:

0, 2i, -2i  
m:1 m:1 m:1



**NO CALCULATOR ALLOWED.**

25.  $f(x) = -x^2(x - 3)^3$

Degree of function: 5

Even or odd degree: odd

Positive or negative leading coefficient:

neg

End Behavior:

As  $x \rightarrow -\infty, f(x) \rightarrow \infty$ .

As  $x \rightarrow \infty, f(x) \rightarrow -\infty$ .

Roots, including multiplicity:

0 m:2 ; 3 m:3

