

No quiz today, we'll have one on
Friday.

Questions on 3.4/3.5 HW?

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2. $x^2 + 4x + 3 = 0$

3. $x^2 - 5x + 6 = 0$

4. $x^2 + 4x = 12$

5. $(x + 4)(x - 3)(x + 1) = 0$

6. $x(x^2 - 6x + 9) = 0$

$x = -4, 3, -1$

$x + 4 = 0$ $x - 3 = 0$ $x + 1 = 0$
 $x = -4$ $x = 3$ $x = -1$

$(-4 + 4)(-4 - 3)(-4 + 1) = 0$
 $0(-7)(-3) = 0$

$(3 + 4)(3 - 3)(3 + 1) = 0$
 $7(0)(4) = 0$

Set
 Topic: Combining polynomial functions.
 Given $f(x) = x^2 + 3x + 2$ and $g(x) = 5x - 4$, find:

7. $f(x) + g(x)$ 8. $f(x) - g(x)$ 9. $f(x) \cdot g(x)$

8.50 x 11.00 in

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5. $(x + 4)(x - 3)(x + 1) = 0$ $0(0-3)(0-3) = 0$

6. $x(x^2 - 6x + 9) = 0$
 $x(x-3)(x-3) = 0$ $x(x-3)^2 = 0$

Set $x = 0, 3, 3$

Topic: Combining polynomial functions.

Given $f(x) = x^2 + 3x + 2$ and $g(x) = 5x - 4$, find:

7. $f(x) + g(x)$ 8. $f(x) - g(x)$ 9. $f(x) \cdot g(x)$

8.50 x 11.00 in

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+ $g(x)$ 8. $f(x) - g(x)$ 9. $f(x) \cdot g(x)$

$(x^2 + 3x + 2)(5x - 4)$

$5x^3 - 4x^2 + 15x^2 - 12x + 10x - 8$

$5x^3 + 11x^2 - 2x - 8$

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18. $(3x^2 + 4x) + (x^2 - 5)$
 $4x^2 + 4x - 5$

19. $4x^2 + 4x^3 + 3x^2 - x + 6$

20. $\frac{6x^3y^5}{9yz^4}$

21. $\left(\frac{2x^{-2}y^3z^4}{yz^4}\right)^2 = \left(\frac{2y^2}{x^2}\right)^2 = \frac{4y^4}{x^4}$

22. $\frac{6x+9}{3}$

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Polynomial Functions 20

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2. $f(n-1) = f(n) + 3; f(1) = 4$
 $(1, 4); (2, 7); (3, 10); (4, 13) \dots$

3. $f(x) = \sqrt{x-3} + 1$ increasing
 no intercepts
 no max/min

4. $f(x) = \log_2 x - 1$

domain: $\{1, 2, 3, \dots\}$
 range: $\{4, 7, 10, \dots\}$

5.

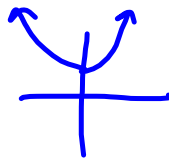
6.

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10.	Equation: $h(x) = (x - 3)(x + 4)(x + 1) = 0$ $x = 3, -4, -1$	Graph:
<p>What I know about this function:</p> <p>x^3 f . . . -</p> <p>degree : 3 , odd , positive leading coeff.</p> <p>End behavior:</p> <p>as $x \rightarrow -\infty$, $h(x) \rightarrow -\infty$</p> <p>as $x \rightarrow \infty$, $h(x) \rightarrow \infty$</p>		
11.	Equation: $f(x) = x^3$	Graph:
<p>What I know about this function:</p> <p>End behavior:</p>		

3.5 The Expansion

A Develop Understanding Task



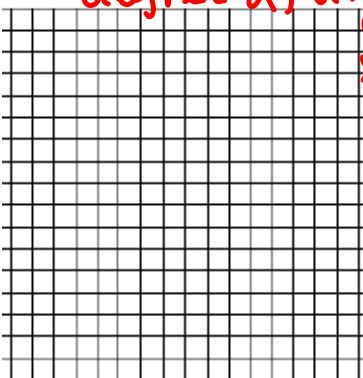
Polynomial functions have interesting characteristics. The degree of the polynomial not only tells us information about the end behavior of the function, it also tells us about the number of roots. In Secondary II, the Fundamental Theorem of Algebra was introduced while studying quadratic functions. The theorem states:

An n^{th} degree polynomial function has n roots.

As we move into polynomials with degree greater than two, let's see if The Fundamental Theorem of Algebra holds true for all polynomial functions. Make a conjecture as to the shape of each function and sketch the graph below.

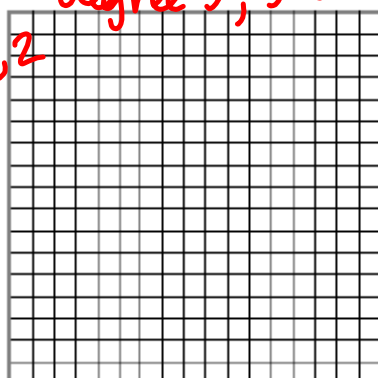
1. $f(x) = (x + 1)(x - 2)$

degree 2, 2 roots @ $x = -1, 2$



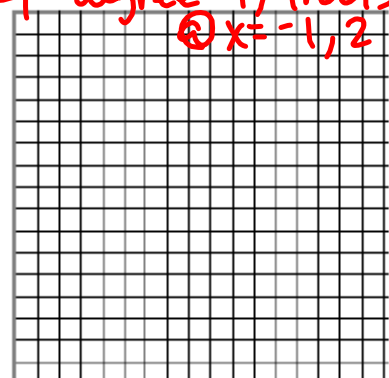
2. $f(x) = x(x + 1)^2$

degree 3; 3 roots @ $x = 0, -1$

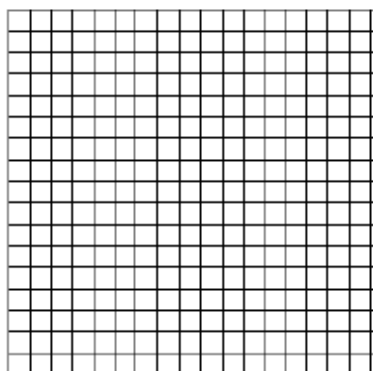


3. $f(x) = (x + 1)^2(x - 2)^2$

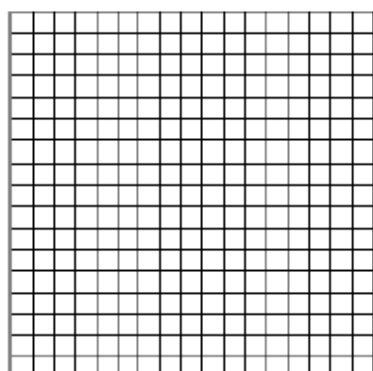
degree: 4; 4 roots @ $x = -1, 2$



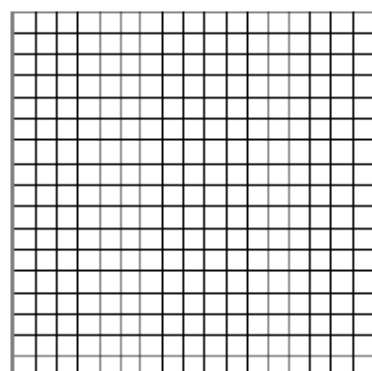
4. $f(x) = x + 1$



5. $f(x) = (x + 1)^2$



6. $f(x) = (x + 1)^3$



7. Once you have made a conjecture about the graphs, confirm your solutions (using technology). How were your conjectures confirmed? What do you need to adjust?

8. Describe the roots of each problem above. How does this match what you know about polynomials and the Fundamental Theorem of Algebra?

5.

$x\text{-int } (-2, 0)$
 $y\text{-int } (0, 4)$
 $\text{dec. } (-\infty, -2)$
 $\text{inc. } (-2, 0)$
 $\text{dec. } (0, 3)$
 $\text{inc. } (3, \infty)$
 $\text{domain: } (-\infty, \infty)$
 $\text{range: } (-5, \infty)$
 no max/min

$$f(x) = \begin{cases} -x^2, & -2 \leq x \leq 2 \end{cases}$$

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Part II: The Binomial Expansion.

9. You may have noticed that questions 4, 5, and 6 are 'similar' in that they contain the same binomial, but are raised to different powers. Question 5 is the same binomial $(x + 1)$ as question 4, but has been expanded to $(x + 1)(x + 1)$ and question 6 has been expanded three times to $(x + 1)(x + 1)(x + 1)$. Rewrite each of the problems in expanded form:

$f(x) = x + 1$

$f(x) = (x + 1)^2$

$f(x) = (x + 1)^3$

Expanded form:

$x + 1$

Expanded form:

$x^2 + 2x + 1$

Expanded form:

$x^3 + 3x^2 + 3x + 1$

$(x + 2y)^6$

The coefficients of the polynomials are written in the table below:

$(x + y)^n$

Pascal's Δ

$(x + 1)^0$	1					
$(x + 1)^1$	1		1			
$(x + 1)^2$	1		2	1		
$(x + 1)^3$	1		3	3	1	
$(x + 1)^4$	1		4	6	4	1
$(x + 1)^5$	1		5	10	10	5
$(x + 1)^6$	1		6	15	20	15
$(x + 1)^n$						

10. Based on the pattern above, what do you think the coefficients would be for $(x + 1)^4$? How about $(x + 1)^5$?

$(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$

$(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$(x + y)^n = cx^n y^0 + cx^{n-1} y^1 + cx^{n-2} y^2 + \dots +$

$+ cx^1 y^{n-1} + cx^0 y^n$

★ 3rd term in expansion of $(v + 4)^3$. $\begin{cases} x = v \\ y = 4 \\ n = 3 \end{cases}$
 → coeff: 1, 3, 3, 1 (Pascal's Δ)

→ $1x^3 y^0 + 3x^2 y^1 + 3x^1 y^2 + 1x^0 y^3$

→ $1v^3 4^0 + 3v^2 4^1 + 3v^1 4^2 + 1v^0 4^3$

$v^3 + 12v^2 + 48v + 64$

11. Describe how you find the coefficients of the binomial expansion for the 'next' expansion?

12. How would the coefficients change for the binomial: $(x + 2)$?

13. How would the coefficients change for the binomial: $(x + y)$?

14. Extension: Do you have a system for how you could find the binomial expansion for *any* binomial raised to the n power?

3.6 Seeing Structure

A Solidify Understanding Task

Claire and Carmella were having a discussion about how easy it is to graph polynomial functions. Claire stated: "All you need to know to sketch the graph of a polynomial function is the degree of the polynomial. The degree will tell you the end behavior and the number of times the graph will cross the x-axis." Carmella mostly agreed, however, thought there was something not quite right with this statement.

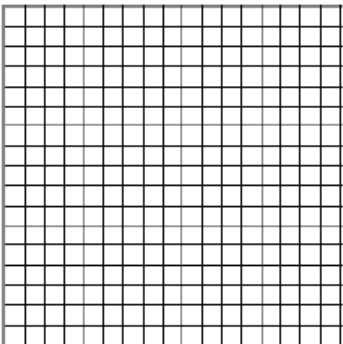


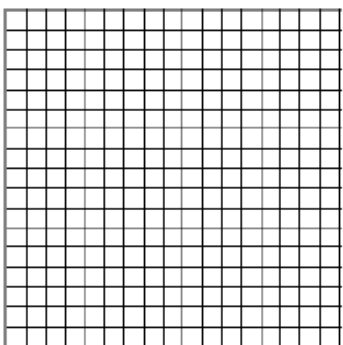
1. Modify Claire's statement about sketching the graph of a polynomial function:

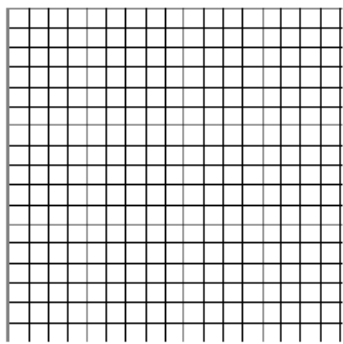
need zeroes/roots;
leading coeff + / - ; multiplicity of each root .

For each function, identify the end behavior and roots (including the multiplicity) of the function.

<p>2. Equation: $f(x) = -x(x-2)(x-4)$</p> <p>degree: 3 lead. coeff: -1</p> <p>Intercepts: $x = 0, 2, 4$ (multiplicity = 1 for all)</p> <p>End behavior: as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$ as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$</p>	<p>Graph:</p>
<p>3. Equation: $f(x) = x(x^2 + 4x + 4)$</p> <p>Intercepts:</p> <p>End behavior: as $x \rightarrow -\infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$ as $x \rightarrow \infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p>	<p>Graph:</p>

4. Equation: $f(x) = g(x) = x^3 - x^2$	Graph:
<p>Intercepts:</p> <p>End behavior:</p> <p>as $x \rightarrow -\infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p> <p>as $x \rightarrow \infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p>	

5. Equation: $f(x) = x^4 - 16$	Graph:
<p>Intercepts:</p> <p>End behavior:</p> <p>as $x \rightarrow -\infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p> <p>as $x \rightarrow \infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p>	

6. Equation: $f(x) = x^3 - 2x^2 - 3x$	Graph:
<p>Intercepts:</p> <p>End behavior:</p> <p>as $x \rightarrow -\infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p> <p>as $x \rightarrow \infty$, $f(x) \rightarrow \underline{\hspace{2cm}}$</p>	

7. Explain how you are able to graph a polynomial that is not already in factored form?

8. If you know one root of a cubic function, can you find the others? Explain?

Homework

3.5 "Ready, Set, Go"