Questions on 2.2?

(1) $\log_5 625 = 4$ $(5^4 = 625)$

(2) $\log_3 243 = 5$ $(3^5 = 243)$

(3) $\log_5 0.2 = -1$ $(5^{-1} = \frac{1}{5})$

(16) $(d^{-3})^{-2} = d^{-3 \cdot -2} = d^6$

$\hookrightarrow d^{-3} \cdot d^{-3}$

(14) $(2^3 w)^4 = 2^{12} w^4$

(17) $x^2 \cdot (x^5)^2 = x^2 \cdot x^{10} = x^{12}$

(18) $m^{-3} (m^2)^3 = m^{-3} \cdot m^6 = m^{-3+6} = m^3$

$\hookrightarrow \frac{m^6}{m^3}$

(19) $x^{-20} \cdot x^{25} = x^5$

(20) $(5a^3)^2 = 5a^3 \cdot 5a^3 = 5^2 a^6$

(22) $(2a^3b^2)^2 = 2^2 a^6 b^4$

Hang on to:

- chapter summaries from textbook
- Modules 4, 1, 2

Mar 28-11:27 AM

2.3 Chopping Logs

A Solidify Understanding Task

Abe and Mary are working on their math homework together when Abe has a brilliant idea!

Abe: I was just looking at this log function that we graphed in Falling Off A Log:

$$y = \log_2(x+b).$$

I started to think that maybe I could just "distribute" the log so that I get:

$$y = \log_2 x + \log_2 b.$$

I guess I'm saying that I think these are equivalent expressions, so I could write it this way:

$$\underbrace{\log_2(x+b)}_{y_1} = \underbrace{\log_2 x + \log_2 b}_{y_2}$$

Mary: I don't know about that. Logs are tricky and I don't think that you're really doing the same thing here as when you distribute a number.

1. What do you think? How can you verify if Abe's idea works?

   No, b cannot be negative for distributing to work. We can graph or choose x & b values to verify.

2. If Abe's idea works, give some examples that illustrate why it works. If Abe's idea doesn't work, give a counter-example.

$x=6$

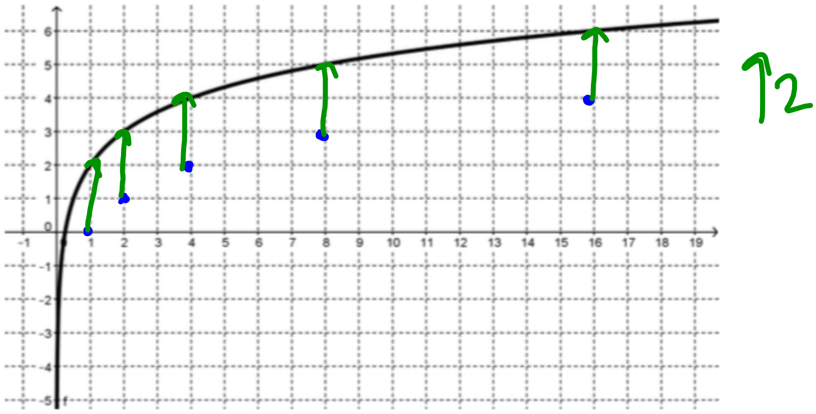
$b=4$

$$\log_2(6+4) \stackrel{?}{=} \log_2 6 + \log_2 4$$

$$\log_2 10 \stackrel{?}{=} 2.58 + 2$$

$$3.32 \neq 4.58$$

Abe: I just know that there is something going on with these logs. I just graphed $f(x) = \log_2(4x)$. Here it is:



It's weird because I think that this graph is just a translation of $y = \log_2 x$. Is it possible that the equation of this graph could be written more than one way?

3. How would you answer Abe's question? Are there conditions that could allow the same graph to have different equations?

$f(x) = 2 + \log_2 x = \log_2(4x)$

$\log_2 4 + \log_2 x = \log_2(4x)$

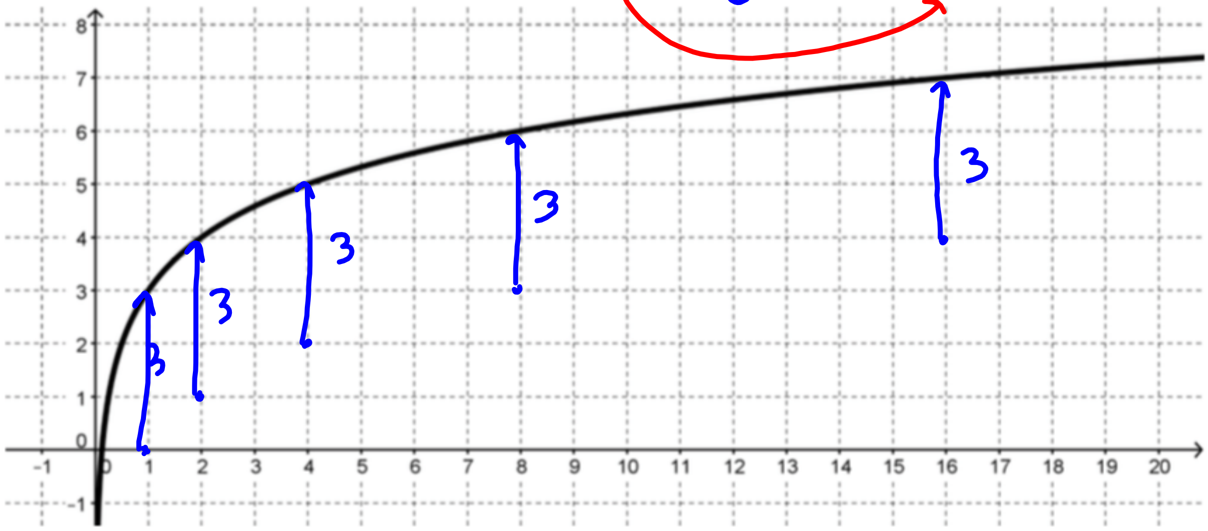
Mary: When you say, "a translation of $y = \log_2 x$" do you mean that it is just a vertical or horizontal shift? What could that equation be?

4. Find an equation for $f(x)$ that shows it to be a horizontal ~~or~~ vertical shift of $y = \log_2 x$.

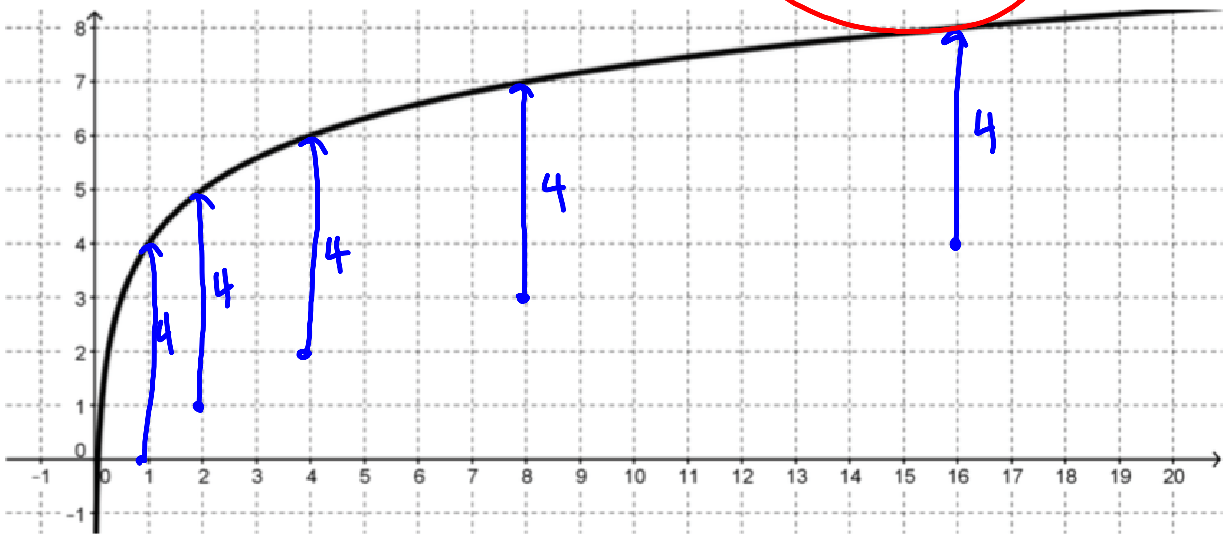
Mary: I wonder why the vertical shift turned out to be up 2 when the x was multiplied by 4. I wonder if it has something to do with the power that the base is raised to, since this is a log function. Let's try to see what happens with $y = \log_2(8x)$ and $y = \log_2(16x)$.

5. Try to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2(8x)$ Equivalent equation: $3 + \log_2 x = \log_2 \boxed{8} + \log_2 \boxed{x}$



b. $y = \log_2(16x)$ Equivalent equation: $4 + \log_2 x = \log_2 \boxed{16} + \log_2 x$



Mary: Oh my gosh! I think I know what is happening here! Here's what we see from the graphs:

$$\log_2(4x) = 2 + \log_2 x$$

$$\log_2(8x) = 3 + \log_2 x$$

$$\log_2(16x) = 4 + \log_2 x$$

$2^4 \cdot 2^2$

Here's the brilliant part: We know that $\log_2 4 = 2, \log_2 8 = 3$, and $\log_2 16 = 4$. So:

$$\log_2(4x) = \log_2 4 + \log_2 x$$

$$\log_2(8x) = \log_2 8 + \log_2 x$$

$$\log_2(16x) = \log_2 16 + \log_2 x$$

I think it looks like the "distributive" thing that you were trying to do, but since you can't really distribute a function, it's really just a log multiplication rule. I guess my rule would be:

$$\log_2(ab) = \log_2 a + \log_2 b$$

6. How can you express Mary's rule in words?

The log of a product becomes the sum of 2 logs with the same base and each factor in one log.

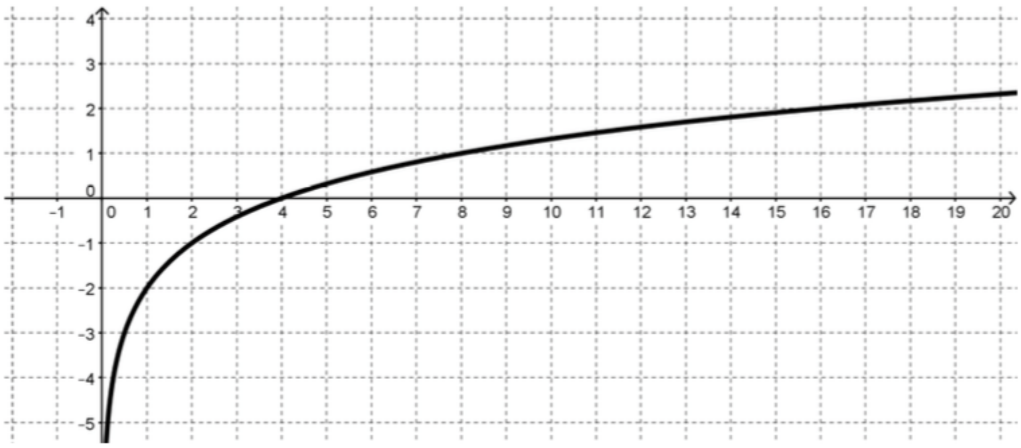
7. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

Mary: So, I wonder if a similar thing happens if you have division inside the argument of a log function. I'm going to try some examples. If my theory works, then all of these graphs will just be vertical shifts of $y = \log_2 x$.

$-2 + \log_2 x =$

8. Here are Abe's examples and their graphs. Test Abe's theory by trying to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2\left(\frac{x}{4}\right)$ Equivalent equation: $\log_2 \frac{1}{4} + \log_2 x =$

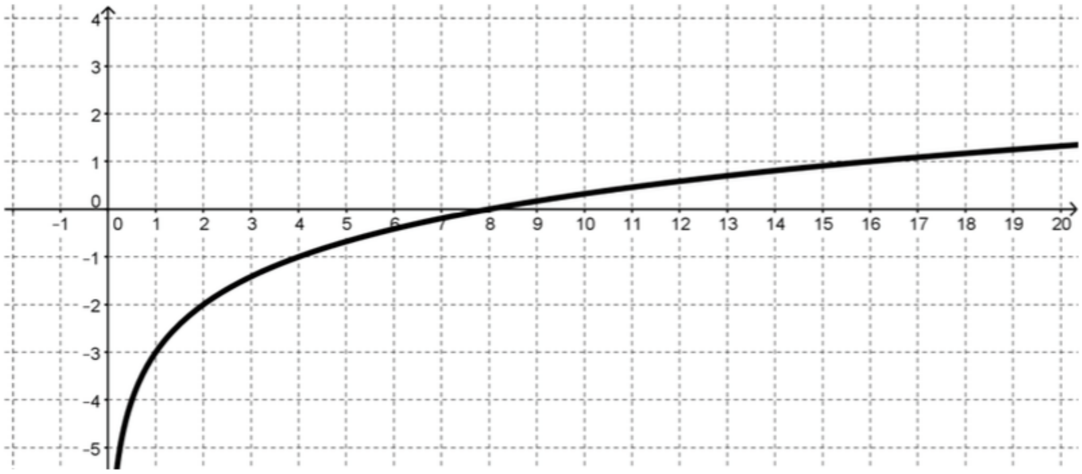


$-2 + \log_2 x$

$-(\log_2 4) + \log_2 x$

$\log_2 x - \log_2 4$

b) $y = \log_2\left(\frac{x}{8}\right)$ Equivalent equation: $\log_2 x - \log_2 8$



9. Use these examples to write a rule for division inside the argument of a log that is like the rule that Mary wrote for multiplication inside a log.

10. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

Abe: You're definitely brilliant for thinking of that multiplication rule. But I'm a genius because I've used your multiplication rule to come up with a power rule. Let's say that you start with:

$$\log_2(x^3)$$

Really that's the same as having: $\log_2(x \cdot x \cdot x)$

So, I could use your multiplying rule and write: $\log_2 x + \log_2 x + \log_2 x$

I notice that there are 3 terms that are all the same. That makes it: $3\log_2 x$

So my rule is: $\log_2(x^3) = 3\log_2 x$

If your rule is true, then I have proven my power rule.

Mary: I don't think it's really a power rule unless it works for any power. You only showed how it might work for 3.

Abe: Oh good grief! Ok, I'm going to say that it can be any number x, raised to any power, k. My power rule is:

$$\left\{ \log_2(x^k) = k\log_2 x \right\}$$

Are you satisfied?

11. Make an argument about Abe's power rule. Is it true or not?

Abe: Before we win the Nobel Prize for mathematics I suppose that we need to think about whether or not these rules work for any base.

12. The three rules, written for any base $b > 1$ are:

Log of a Product Rule: $\log_b(xy) = \log_b x + \log_b y$

Log of a Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Log of a Power Rule: $\log_b(x^k) = k \log_b x$

→ use for 13-24 on pg. 24-25

Make an argument for why these rules will work in any base $b > 1$ if they work for base 2.

13. How are these rules similar to the rules for exponents? Why might exponents and logs have similar rules?

Homework

Finish 2.3 "Ready, Set, Go"