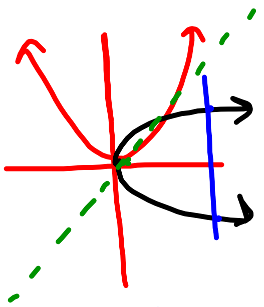
Starter

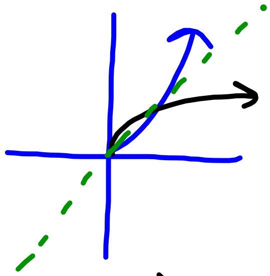
Get out a piece of paper and write down everything you remember about inverse functions.

Inverse Functions

- x & y axes switch
- slopes of linear inverses are reciprocals
- y-intercept becomes x-intercept
  $(0, a) \longrightarrow (a, 0)$
- domain & range switch



Quadratic functions have inverses when we restrict the domain b/c if we don't restrict the domain, the inverse is NOT a function.



- Inverses are reflections across $y = x$
- $(x, y) \rightarrow (y, x)$

2.1 Log Logic

A Develop Understanding Task

We began thinking about logarithms as inverse functions for exponentials in Tracking the Tortoise. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:

Input Output

$x = 3$ → $2^3 = 8$ → 3

$f(x) = 2^x$ $\boxed{f^{-1}(x) = \log_2 x}$

We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$ what power of 2 is 8?

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\log_{10} 100 = 2$

$10^3 = 1000$ so, $\log_{10} 1000 = 3$

The notation is a little strange, but you can see the inverse pattern of switching the inputs and outputs.

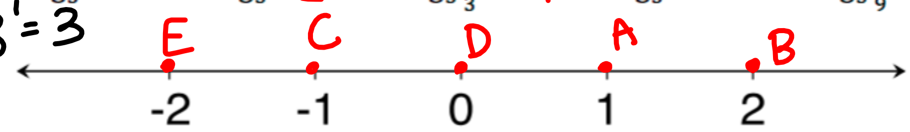
The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns that you may notice with logarithms.

Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

$3^2=9$

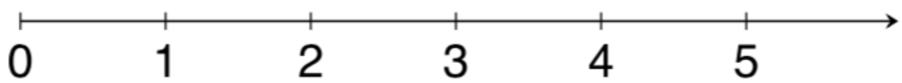
1. A. $\log_3 3 = 1$ B. $\log_3 9 = 2$ C. $\log_3 \frac{1}{3} = -1$ D. $\log_3 1 = 0$ E. $\log_3 \frac{1}{9} = -2$

$3^1 = 3$



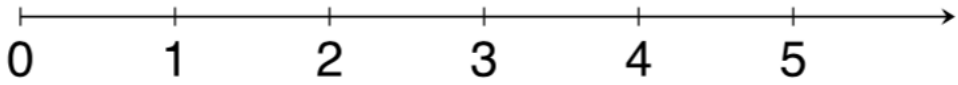
Explain: ____

2. A. $\log_3 81$ B. $\log_{10} 100$ C. $\log_8 8$ D. $\log_5 25$ E. $\log_2 32$



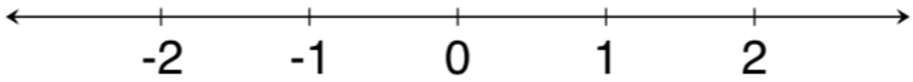
Explain: ____

3. A. $\log_7 7$ B. $\log_9 9$ C. $\log_{11} 1$ D. $\log_{10} 1$



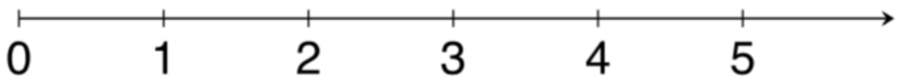
Explain: ____

4. A. $\log_2\left(\frac{1}{4}\right)$ B. $\log_{10}\left(\frac{1}{1000}\right)$ C. $\log_5\left(\frac{1}{125}\right)$ D. $\log_6\left(\frac{1}{6}\right)$



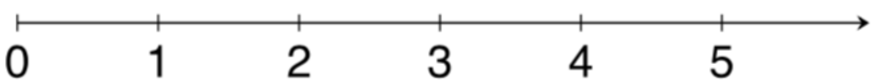
Explain: ____

5. A. $\log_4 16$ B. $\log_2 16$ C. $\log_8 16 = 1.3$ D. $\log_{16} 16$



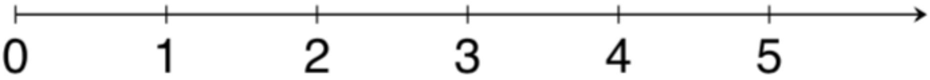
Explain: ____

6. A. $\log_2 5$ B. $\log_5 10$ C. $\log_6 1$ D. $\log_5 5$ E. $\log_{10} 5$



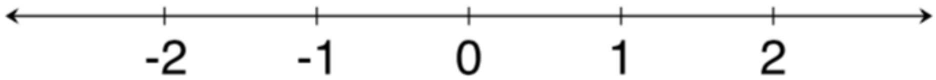
Explain: ____

7. A. $\log_{10}50$ B. $\log_{10}150$ C. $\log_{10}1000$ D. $\log_{10}500$



Explain: ______________________________

8. A. $\log_3 3^2 = 2$ B. $\log_5 5^{-2} = -2$ C. $\log_6 6^0 = 0$ D. $\log_4 4^{-1} = -1$ E. $\log_2 2^3 = 3$



Explain: ______________________________

Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of $\log_b x$ is positive.

Explain: Sometimes true; negative when x is a proper fraction.

$y = \log_b x$

10. $\log_b x$ is not a valid expression if x is a negative number.

Explain: Always true; when $b^y = x$, x can never be negative if b is positive.

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11. $\log_b 1 = 0$ for any base, b > 1.

Explain: Always; $b^0 = 1$ always

12. $\log_b b = 1$ for any b > 1.

Explain: Always; $b^1 = b$ always

$\log_2 1 = \log_3 1$

$\log_2 4 > \log_3 4$

13. $\log_2 x < \log_3 x$ for any value of *x*.

Explain: Never;

14. $\log_b b^n = n$ for any b > 1.

Explain: Always; see below

$\log_b b^{n} =$

$n \underbrace{\log_b b} =$

$n \cdot 1 =$

n

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Homework

Finish 2.1 "Ready, Set, Go"