Questions on Review #5 WKS?

13, 17, 18

Review #6: Slope Fields and Differential Equations

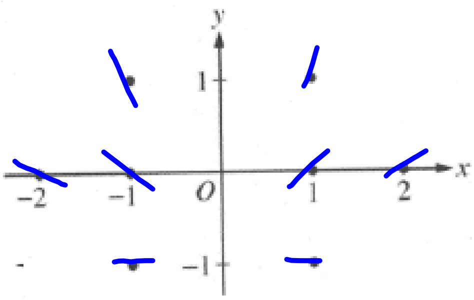
Slope fields: A field of slopes.
Being able to use differential equations to find the slope at any (x,y) point, we can draw a small piece of linearization at that point, which (b/c of local linearity) approximates the solution curve that passes through that point.

EXAMPLE

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



Differential equations:

An equation involving a derivative. The order of a differential equation is the order of the highest derivative involved in the equation.

* A differential equation has general solutions and particular solutions, when given an initial condition.

Solving a differential equation:

1- Separate variables
y & dy on (L)
x & dx, coeff. on (R)
• NEVER have dy or dx in denominator

2- Integrate both sides.

3- +C on x-side

4- Solve for C using initial condition.

5- Solve for y.

6- If necessary, use initial conditions to choose the correct equation.

EXAMPLES

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

$$\frac{\cancel{x^2}\,dy}{(y-1)\,\cancel{x^2}} = \frac{\cancel{(y-1)}\,dx}{\cancel{(y-1)}\,x^2}$$

$2^4 \cdot 2^3 = 2^{3+4}$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2} \rightarrow \int x^{-2}\,dx$$

$$\ln|y-1| = \frac{x^{-1}}{-1} + C$$

$$e^{\ln|y-1|} = e^{\left(-\frac{1}{x}+C\right)}$$

$$|y-1| = e^{-1/x} \cdot e^{c}$$

$k = \pm e^{c}$ (takes care of $|y-1|$)

$$y-1 = ke^{-1/x}$$

(2,0) $\quad 0-1 = ke^{-1/2}$

$$\frac{-1}{e^{-1/2}} = \frac{k\cancel{e^{-1/2}}}{\cancel{e^{-1/2}}}$$

$$\underline{-e^{1/2} = k} \quad \text{(our C)}$$

$$y-1 = -e^{1/2} \cdot e^{-1/x}$$

$$\boxed{y = 1 - e^{(1/2 - 1/x)}}$$

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

Applications: Population equations:

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$$

a) If $P(0) = 3$, what is $\lim_{t \to \infty} P(t)$? If $P(0) = 20$, what is $\lim_{t \to \infty} P(t)$?

b) If $P(0) = 3$, for what valaue of P is the population growing the fastest?

Apr 7-2:43 PM