Questions on HW?

Review #5: Definite and Indefinite Integrals

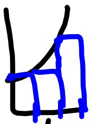
An integral is . . .

- area under a curve
- antiderivative

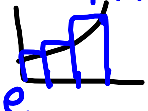
$$A = \int_a^b f(x)\,dx$$

Estimating an integral:

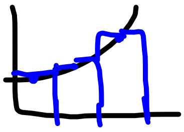
LRAM (left rect. approx. method) - under approx.



RRAM (right " " ") - over approx.



MRAM (midpoint " " ") - most accurate



EXAMPLE

3.5 ↓

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, when time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

$$H(3.5) \approx \frac{H(5)-H(2)}{5-2} = \frac{52-60}{3} = -\frac{8}{3} = -2.\overline{6} \text{ or } -2.7°C$$

per minute

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

The average temperature of the tea, in degrees Celsius, over the ten minutes.

$$\frac{1}{10}\int_0^{10} H(t)dt = \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right) =$$

$$= \boxed{52.95}$$

The definite integral:

If $y=f(x)$ is nonnegative & integrable over a closed interval, $[a,b]$, then the area under the curve $y=f(x)$ from a to b is the integral of f from a to b,

$$A=\int_a^b f(x)dx$$

Fundamental Theorem of Calculus, Part 2:

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)dx = F(b)-F(a)$$

(also called the Integral Evaluation Thm.)

Rules for integration:

Rules (pg.289)

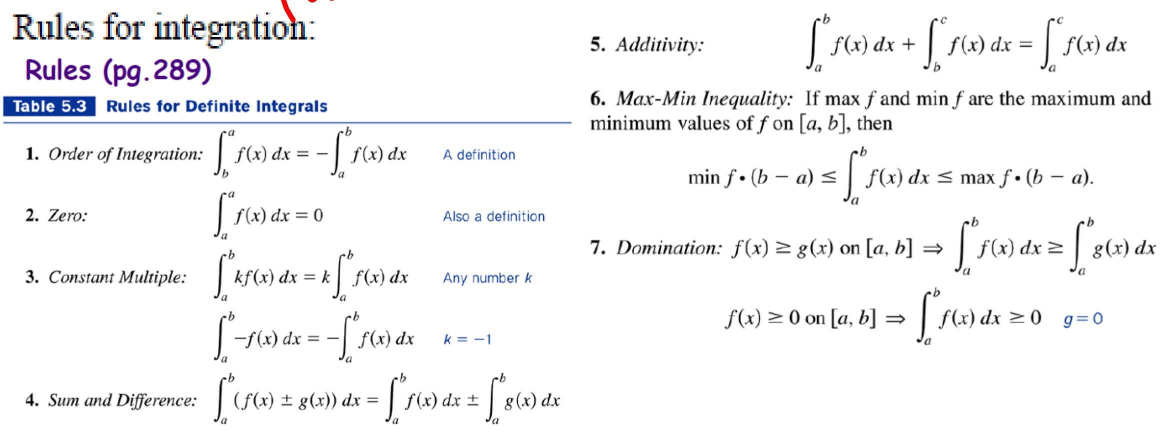


Table 5.3 **Rules for Definite Integrals**

1. *Order of Integration:*	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A definition
2. *Zero:*	$\int_a^a f(x)\,dx = 0$	Also a definition
3. *Constant Multiple:*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any number k
	$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$	$k=-1$
4. *Sum and Difference:*	$\int_a^b (f(x)\pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f\cdot(b-a) \le \int_a^b f(x)\,dx \le \max f\cdot(b-a).$$

7. *Domination:* $f(x)\ge g(x)$ on $[a,b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

$$f(x)\ge 0 \text{ on } [a,b] \Rightarrow \int_a^b f(x)\,dx \ge 0 \quad g=0$$

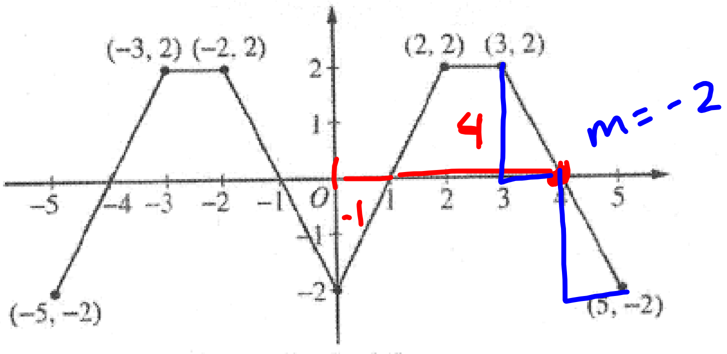
Average value of a function:

If f is integrable on $[a,b]$, its average (mean) value on $[a,b]$ is

$$av(f)=\frac{1}{b-a}\int_a^b f(x)\,dx$$

You are likely to see this type of problem on the AP test:

$g'(x)=f(x)$



Graph of f

. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x=1$? Justify your answer.

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5)=2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x=108$.

ⓐ $g(4)=\int_0^4 f(t)dt = 4+-1=3$ $g''(4)=f'(4)=-2$

$g'(4)=f(4)=0$

ⓑ $-\ \circ\ +$ $g'=f$

$\xleftrightarrow{\quad 1 \quad}$

g has a relative minimum at x=1 because $g'=f$ changes from negative to positive at $x=1$.

ⓒ $g(5)=2$

$g(10)=2\cdot g(5)=4$

$x=5,10,15,\dots,100,105,110$

21 periods

$g(108)=\int_0^{105} f(t)dt+\int_{105}^{108} f(t)dt$

$=21\cdot g(5)+g(3)$

$=21\cdot 2+2$

$=44$

$(108,44)$

$\boxed{y-44=\frac{2}{5}(x-108)}$

Fundamental Theorem of Calculus, Part 1: If f is continuous on $[a,b]$, then the function $F(x)=\int_a^x f(t)\,dt$ has a derivative at every point x in $[a,b]$, and $\frac{dF}{dx}=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$

For example: $\frac{d}{dx}\int_{-\pi}^{x}\cos t\,dt=\cos x$

Indefinite integrals you must know: 7.1 notes

$\int u^n\,du=\frac{u^{n+1}}{n+1}+C$, when $n\neq -1$

$\int a^u \ln a\,du=\frac{a^u}{\ln a}+C$

$\int e^u\,du=e^u+C$

$\int \sin u\,du=-\cos u+C$

$\int \cos u\,du=\sin u+C$

$\int \sec^2 u\,du=\tan u+C$

$\int \csc^2 u\,du=-\cot u+C$

$\int \sec u\tan u\,du=\sec u+C$

$\int \csc u\cot u\,du=-\csc u+C$

$\int \tan u\,du=-\ln|\cos u|+C$

$\int \cot u\,du=\ln|\sin u|+C$

$\int \frac{du}{u}=\ln|u|+C$

$\int \frac{du}{\sqrt{1-u^2}}=\arcsin u+C$

$\int \frac{du}{1+u^2}=\arctan u+C$

$\int \frac{du}{u\ln a}=\log_a u+C$

Tricks:

Simplify, factor, rewrite

Integration strategies: *u*-substitutions

$$\frac{1}{6}\int \overbrace{\boxed{x^2}\sqrt{\underbrace{5+2x^3}_{u}}\,\boxed{dx}}^{du}=\frac{1}{6}\int\sqrt{u}\,du=\frac{1}{6}\int u^{1/2}du$$

$u=5+2x^3$

$du=6x^2dx$

$\frac{1}{6}du=x^2dx$

$$\frac{1}{6}\cdot u^{3/2}\cdot\frac{2}{3}+C=$$

$$\frac{2}{18}u^{3/2}+C=$$

$$\frac{1}{9}(5+2x^3)^{3/2}+C$$

or

$$\frac{1}{9}\sqrt[3]{(5+2x^3)^2}+C$$