Get out your Review #2 WKS. We will spend 45 mins in class working on it and then 45 mins moving into our Review #3.

From last time...

Find the first and second derivatives of the function $x^3 + 3xy - y^2 = 8$.

$$3x^2 + 3(x \cdot y' + y \cdot 1) - 2y \cdot y' = 0$$

$$3x^2 + 3xy' + 3y - 2y \cdot y' = 0$$

$$3x^2 + 3y = 2y \cdot y' - 3xy'$$

$$\frac{3x^2 + 3y}{2y - 3x} = \frac{y'(2y - 3x)}{2y - 3x}$$

$$\boxed{\frac{3x^2 + 3y}{2y - 3x} = y'}$$

$$\frac{(2y - 3x)(6x + 3y') - (3x^2 + 3y)(2y' - 3)}{(2y - 3x)^2} = y''$$

$$\frac{12xy + 6yy' - 18x^2 - 9xy' + (-6x^2y' + 9x + -6yy' + 9y)}{(2y - 3x)^2} = y''$$

$$\frac{12xy - 18x^2 - 9xy' - 6x^2y' + 9x + 9y}{(2y - 3x)^2} = y''$$

$$\boxed{\frac{12xy - 18x^2 - 9x\left(\frac{3x^2 + 3y}{2y - 3x}\right) - 6x^2\left(\frac{3x^2 + 3y}{2y - 3x}\right) + 9x + 9y}{(2y - 3x)^2} = y''}$$

Let *f* be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If a tangent to the graph at $x=3$ is used to find an approximation of a zero of *f*, what is that approximation?

Write the equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at $x=1$.

Review #3

First derivative: tells where $f(x)$ is increasing (+) & decreasing (−)

Second derivative: tells where $f(x)$ is concave (+) ↑ and (−) ↓

Graph	Above x-axis indicates	Below x-axis indicates	0 indicates
$f'(x)$	$f(x)$ is increasing	$f(x)$ is decreasing	$f(x)$ has a horizontal tangent
$f''(x)$	$f(x)$ is concave ↑	$f(x)$ is concave ↓	point of inflection

To find where $f(x)$ is increasing or decreasing:

$f(x)$ inc: $f'(x)$ (+) & so will graph

$f(x)$ dec: $f'(x)$ (−) & so will graph

Justifying your answer:

Specific & concise

$f(x)$ is inc. ______ b/c $f'(x) > 0$

$f(x)$ is dec. ______ b/c $f'(x) < 0$

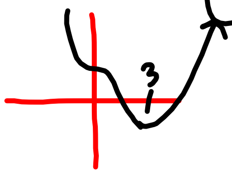
Mar 31-2:40 PM

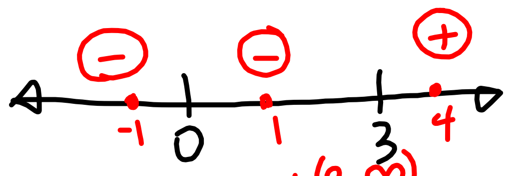
EXAMPLE: On what intervals is the function $f(x) = x^4 - 4x^3 + 10$ increasing or decreasing?

$f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

$x = 0, 3$





Increasing: $(3, \infty)$
Decreasing: $(-\infty, 3)$

To find where $f(x)$ is concave up or concave down:

$f''(x) > 0$, concave ↑

$f''(x) < 0$, concave ↓

Justifying your answer:

Specific & concise:

$f(x)$ is concave up ____ because $f''(x) > 0$

" " down ____ " $f''(x) < 0$

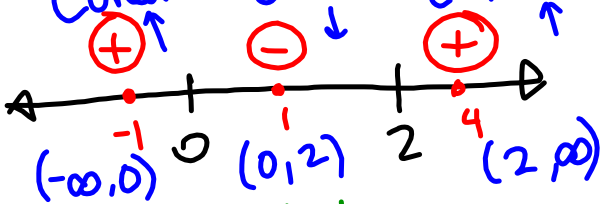
Find all intervals on which the function $f(x) = x^4 - 4x^3 + 10$ is concave up or concave down.

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

$x = 0, 2$



Global/absolute maximum/minimum:
↓
local

* pg. 209-210 has good visuals *

Absolute max/min: absolute highest/lowest y-value

local/global max/min: max/min for an interval.

To find:

Check critical points where $f'(x) = 0$ (and endpoints)

global max → If f' changes from ⊕ to ⊖

global min → If f' changes from ⊖ to ⊕

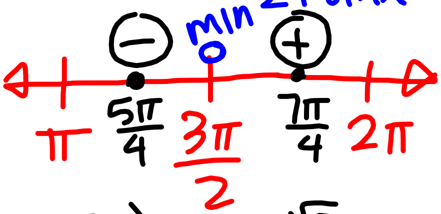
EXAMPLES

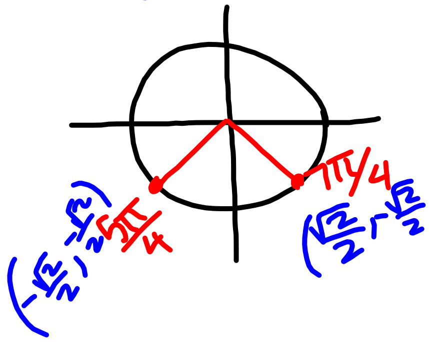
1. (No calculator) Let f be the function defined by $f(x) = \ln(2+\sin x)$ for $\underbrace{\pi \le x \le 2\pi}_{}$ end points. Find the absolute maximum value and the absolute minimum value of f. Show the work that leads to your answer.

$$f'(x) = \frac{1}{2+\sin x} \cdot \cos x = \frac{\cos x}{2+\sin x}$$

$f'(x)=0$ where $\cos x = 0$ at $x = \cancel{\frac{\pi}{2}}$, $\frac{3\pi}{2}$

not in domain





$$f'\left(\frac{5\pi}{4}\right) = \frac{-\frac{\sqrt{2}}{2}}{2+-\frac{\sqrt{2}}{2}} = \frac{-}{+}$$

$$f'\left(\frac{7\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{2-\frac{\sqrt{2}}{2}} = \frac{+}{+}$$

$f(\pi) = 0. \ldots\ldots$

$f\left(\frac{3\pi}{2}\right) =$ abs. min

$f(2\pi) = 1. \ldots\ldots$ abs. max

Finish Wednesday...

2. (Calculator allowed) The rate at which people enter an amusement park on a given day is modeled by $E(t) = \frac{15{,}600}{t^2 - 24t + 160}$. The rate at which the people leave the same amusement park the same day is modeled by $L(t) = \frac{9890}{t^2 - 38t + 370}$. T is measured in hours after midnight, and the functions are valid for $9 \le t \le 23$. At 9 a.m. ($t = 9$) there are no people in the park. At what time t is the number of people in the park a maximum?

Local/relative maxima/minima:

To find:

EXAMPLES

3. (No calculator) Find all local extrema for the function $f(x) = 2xe^{-x}$ on the interval $(0, \infty)$.

4. (Calculator allowed) Find all local extrema for the function $y = e^x - 3x^2$.

Local extrema from a graph of f'.

Finding points of inflection:

Position, velocity, acceleration:

(Calculator)

A particle moves along a line in such a way that at time t, $1 \le t \le 8$, its position is given by

$$s(t) = \int_1^t \left[1 - x\cos t - (\ln x)(\sin x)\right] dx$$

(a) Write the formula for the velocity of the particle at time t.

(b) At what instant does the particle reach its maximum speed?

(c) When is the particle moving to the left?