I will check off your two worksheet packets from last week, so get those ready.

Questions on Unit 1 Review WKS?

9 $f(x) = x^2 - 1 \rightarrow \lim_{x\to 1} \frac{f(x+1) - f(2)}{x^2-1}$

$f(x+1) = (x+1)^2 - 1$
$f(x+1) = x^2 + 2x + 1 - 1$
$f(x+1) = x^2 + 2x$

$f(2) = 2^2 - 1$
$f(2) = 3$

$\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1} =$

$\lim_{x\to 1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} =$

$\lim_{x\to 1} \frac{(x+3)}{(x+1)} = \frac{1+3}{1+1} =$

$\frac{4}{2} = 2$

C

6 $\lim_{x\to\infty} \frac{3x^3-5x}{1x^3-2x^2+1} = 3$

Horizontal Asy @ $y=3$

Unit 1 Review - Quickly fill in if needed...

Limits - Informal definition:

One-sided limits:

Theorem:

Evaluating limits algebraically:

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Limits involving infinity:

Limits which are infinite:

Limits as $n \to \pm\infty$:

Evaluating limits logarithmically:

A function is continuous if . . .

Types of discontinuities:

Use the definition to determine whether or not the following function is continuous at the given points.

$$f(x)=\begin{cases} \sqrt{x-3}, & 3 \le x < 7 \\ x-5, & x \ge 7 \end{cases}$$

Is f continuous at $x=4$?

Is f continuous at $x=7$?

Properties of continuous functions:

Intermediate Value Theorem:

Extreme Value Theorem:

~~Unit~~ #2 Review
units 3 & 4

Derivatives:

The derivative of the function f with respect to x is the function f'(x) whose value at x is

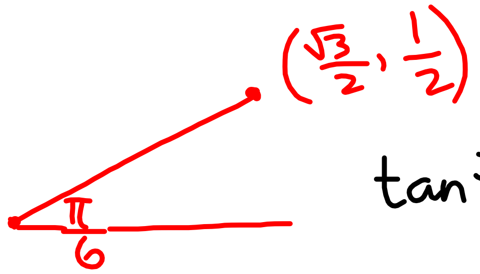
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLES

$f(x) = \tan x, \; x = \frac{\pi}{6}$

1. $\lim_{h \to 0} \dfrac{\tan\left(\frac{\pi}{6} + h\right) - \frac{1}{\sqrt{3}}}{h} =$

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$



$$\tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

2. $\lim_{h \to 0} \frac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h} =$

(A) $\frac{5}{2}$ (B) $\frac{5}{16}$ (C) 40 (D) 160 (E) DNE

$f(x) = 5x^4$

$x = \frac{1}{2}$

$5\left(\frac{1}{2}\right)^4$

$5 \cdot \frac{1}{16}$

A function f is differentiable at $x = c$ if and only if . . .

f is continuous at $x=c$

Rules for differentiation:

Sum, difference, power, constant multiple, quotient, product

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15 derivatives you must know: (u is a function of x)

<u>Fifteen derivatives you must know</u> (u and v are functions of x; a is a number)

Wed. 4/5
Quiz

$f(x)=u^a$	$f'(x)=au^{a-1}\cdot u'$
$f(x)=e^u$	$f'(x)=e^u\cdot u'$
$f(x)=a^u$	$f'(x)=a^u\cdot \ln a\cdot u'$
$f(x)=\ln u$	$f'(x)=\frac{1}{u}\cdot u'=\frac{u'}{u}$
$f(x)=\log_a u$	$f'(x)=\frac{1}{u\cdot \ln a}\cdot u'=\frac{u'}{u\cdot \ln a}$
$f(x)=\sin u$	$f'(x)=\cos u\cdot u'$
$f(x)=\cos u$	$f'(x)=-\sin u\cdot u'$
$f(x)=\tan u$	$f'(x)=\sec^2 u\cdot u'$
$f(x)=\cot u$	$f'(x)=-\csc^2 u\cdot u'$
$f(x)=\sec u$	$f'(x)=\sec u\cdot \tan u\cdot u'$
$f(x)=\csc u$	$f'(x)=-\csc u\cdot \cot u\cdot u'$
$f(x)=\arcsin u$	$f'(x)=\frac{1}{\sqrt{1-u^2}}\cdot u'=\frac{u'}{\sqrt{1-u^2}}$
$f(x)=\arccos u$	$f'(x)=-\frac{1}{\sqrt{1-u^2}}\cdot u'=-\frac{u'}{\sqrt{1-u^2}}$
$f(x)=\arctan u$	$f'(x)=\frac{1}{1+u^2}\cdot u'=\frac{u'}{1+u^2}$
$f(x)=\text{arc}\cot u$	$f'(x)=-\frac{1}{1+u^2}\cdot u'=-\frac{u'}{1+u^2}$

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Higher-order derivatives:

1st, 2nd, so on . . .

If $f(x) = 2x^3 + 3\sin x - e^x$, find $f''(x)$.

$f'(x) = 6x^2 + 3\cos x - e^x$

$f''(x) = 12x - 3\sin x - e^x$

Implicit differentiation: (4.2 in bk)

1. Differentiate both sides of eqn. w/ respect to x.
2. Collect terms w/ $\frac{dy}{dx}$ on one side of eqn.
3. Factor out $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

What is the slope of the tangent to the curve $y + 2 = \frac{x^2}{2} - 2\sin y$ at the point (2, 0)?

$y' + 0 = \frac{1}{2} \cdot 2x - 2(\cos y)y'$

$y' = x - 2(\cos y)y'$

$y' + 2(\cos y)y' = x$

$\frac{y'(1 + 2\cos y)}{1 + 2\cos y} = \frac{x}{1 + 2\cos y}$

$\boxed{y' = \frac{x}{1 + 2\cos y}}$

(2, 0)

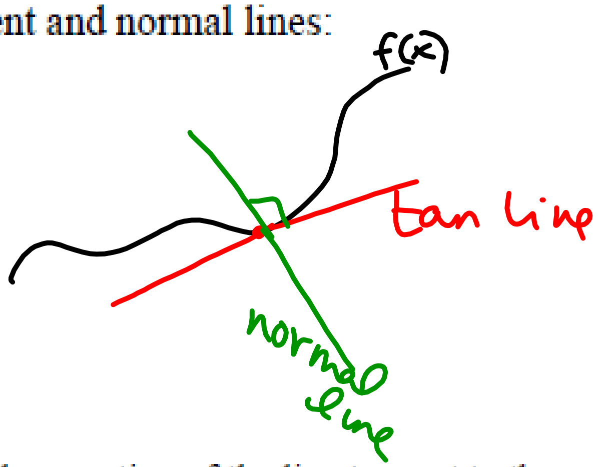
$y' = \frac{2}{1 + 2\cos 0}$

$= \frac{2}{1 + 2} = \frac{2}{3}$

The Mean Value Theorem for derivatives:

If $y=f(x)$ is continuous at every point of the closed interval $[a,b]$ and diff. at every point of (a,b), then there is at least one point c in (a,b) at which $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Tangent and normal lines:



normal lines are $\perp$ to tan lines (neg. reciprocal slope)

Write the equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at $x=1$.

finish Monday...

Linearization:

basically a tan line

$$L(x) = f(a) + f'(a)(x - a)$$

point: $(a, f(a))$
slope: $f'(a)$

Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If a tangent to the graph at $x = 3$ is used to find an approximation of a zero of f, what is that approximation?

finish Monday...

Find the first and second derivatives of the function $x^3 + 3xy - y^2 = 8$.

$$3x^2 + 3(x\cdot y' + y\cdot 1) - 2y\cdot y' = 0$$

$$3x^2 + 3xy' + 3y - 2y\cdot y' = 0$$

$$3x^2 + 3y = 2y\cdot y' - 3xy'$$

$$\frac{3x^2+3y}{2y-3x} = \frac{y'(\cancel{2y-3x})}{\cancel{2y-3x}}$$

$$\boxed{\frac{3x^2+3y}{2y-3x} = y'}$$

$$\frac{(2y-3x)(6x+3y') - (3x^2+3y)(2y'-3)}{(2y-3x)^2} = y''$$

$$\frac{12xy + 6yy' - 18x^2 - 9xy' + (-6x^2y' + 9x + -6yy' + 9y)}{(2y-3x)^2} = y''$$

finish Monday...