

Get out your books and work on finishing 1.2, pages 11-13 with your group. I will be checking off your 1.1 HW (pg.7-9) soon!

## 1.2 Flipping Ferraris

### *A Solidify Understanding Task*

When people first learn to drive, they are often told that the faster they are driving, the longer it will take to stop. So, when you're driving on the freeway, you should leave more space between your car and the car in front of you than when you are driving slowly through a

neighborhood. Have you ever wondered about the relationship between how fast you are driving and how far you travel before you stop, after hitting the brakes?



1. Think about it for a minute. What factors do you think might make a difference in how far a car travels after hitting the brakes?

*force/inertia, angle of road, tire condition, road condition,*

There has actually been quite a bit of experimental work done (mostly by police departments and insurance companies) to be able to mathematically model the relationship between the speed of a car and the braking distance (how far the car goes until it stops after the driver hits the brakes).

**exponential function**

2. Imagine your dream car. Maybe it is a Ferrari 550 Maranello, a super-fast Italian car. Experiments have shown that on smooth, dry roads, the relationship between the braking distance ( $d$ ) and speed ( $s$ ) is given by  $d(s) = 0.03s^2$ . Speed is given in miles/hour and the distance is in feet.
  - a) How many feet should you leave between you and the car in front of you if you are driving the Ferrari at 55 mi/hr?

$$d(55) = 0.03(55)^2$$

$$d(55) = 90.75 \text{ ft} \quad (55, 90.75)$$

- b) What distance should you keep between you and the car in front of you if you are driving at 100 mi/hr?

$$d(100) = 0.03(100)^2$$

$$d(100) = 300 \text{ ft.}$$

(100, 300)

- c) If an average car is about 16 feet long, about how many car lengths should you have between you and that car in front of you if you are driving 100 mi/hr?

$$\frac{300}{16} = 18.75$$

19 car lengths

- d) It makes sense to a lot of people that if the car is moving at some speed and then goes twice as fast, the braking distance will be twice as far. Is that true? Explain why or why not.

No, it will be 4x as far.

s

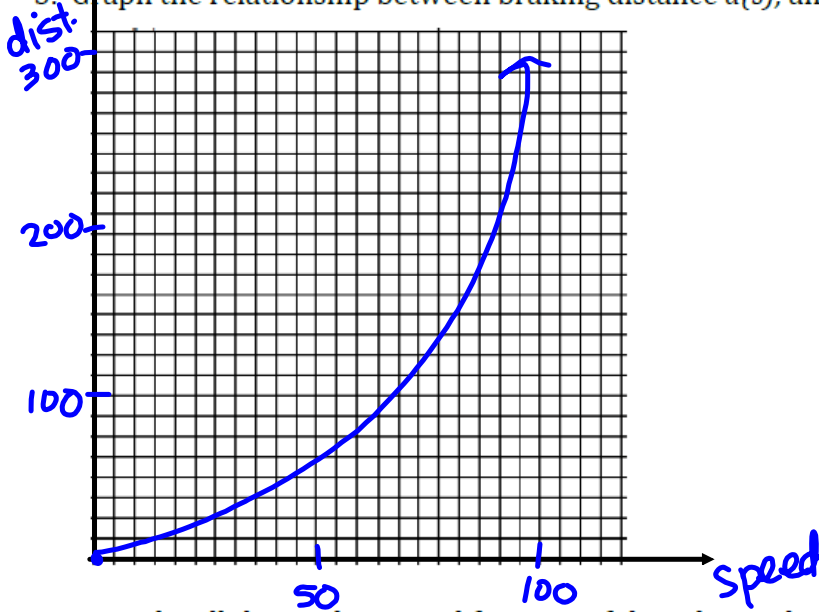
$$d(s) = 0.03s^2$$

2s

$$d(2s) = 0.03(2s)^2$$

$$d(2s) = 0.03(4)s^2$$

3. Graph the relationship between braking distance  $d(s)$ , and speed ( $s$ ), below.



$s$	$d(s)$
0	0
55	90.75
100	306

$$0.03s^2$$

4. Describe all the mathematical features of the relationship between braking distance and speed for the Ferrari modeled by  $d(s) = 0.03s^2$ .

domain:  $[0, \infty)$   
 range:  $[0, \infty)$

Increasing  
 continuous  
 quadratic

5. What if the driver of the Ferrari 550 was cruising along and suddenly hit the brakes to stop because she saw a cat in the road? She skidded to a stop, and fortunately, missed the cat. When she got out of the car she measured the skid marks left by the car so that she knew that her braking distance was 31 ft.

a) How fast was she going when she hit the brakes?

$$\frac{31}{0.03} = \frac{0.03s^2}{0.03}$$

$$\sqrt{1033.\bar{3}} = \sqrt{s^2}$$

$$32.1 = s$$

mph

c) If she didn't see the cat until she was 15 feet away, what is the fastest speed she could be traveling before she hit the brakes if she wants to avoid hitting the cat?

$$15 = 0.03s^2$$

$$22.4 = s$$

mph

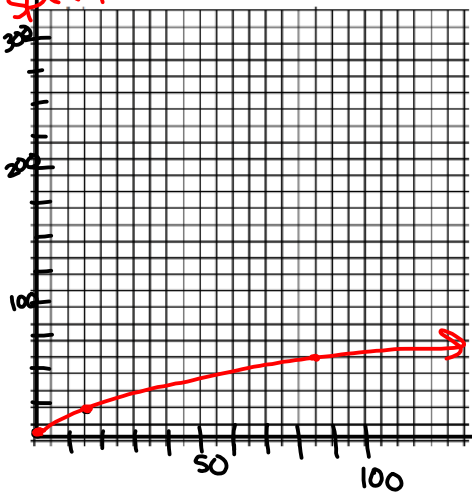
6. Part of the job of police officers is to investigate traffic accidents to determine what caused the accident and which driver was at fault. They measure the braking distance using skid marks and calculate speeds using the mathematical relationships just like we have here, although they often use different formulas to account for various factors such as road conditions. Let's go back to the Ferrari on a smooth, dry road since we know the relationship. Create a table that shows the speed the car was traveling based upon the braking distance.

$d(s)$	$s(d)$
0	0
15	22.4
31	32.1
90.75	55
300	100

7. Write an equation of the function  $s(d)$  that gives the speed the car was traveling for a given braking distance.

$$s(d) = \sqrt{\frac{d}{0.03}}$$

8. Graph the function  $s(d)$  and describe its features.



$$\sqrt{\frac{d}{0.03}}$$

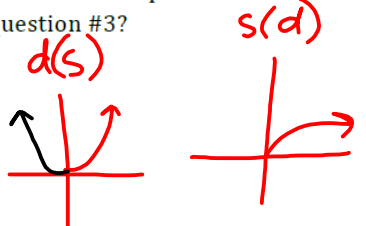
$d(s)$	$s(d)$
0	0
15	22.4
31	32.1
90.75	55
300	100

Mathematics Vision Project | MVP

9. What do you notice about the graph of  $s(d)$  compared to the graph of  $d(s)$ ? What is the relationship between the functions  $d(s)$  and  $s(d)$ ?

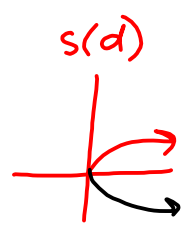
inverses  
 axes switch  
 domain & range switch  
 $(x, y) \rightarrow (y, x)$   
 points switch

10. Consider the function  $d(s) = 0.03s^2$  over the domain of all real numbers, not just the domain of this problem situation. How does the graph change from the graph of  $d(s)$  in question #3?



other half of parabola

11. How does changing the domain of  $d(s)$  change the graph of the inverse of  $d(s)$ ?



12. Is the inverse of  $d(s)$  a function? Justify your answer.

No, it is not a function when the domain of  $d(s)$  is extended to all real #s. It does not pass the vertical line test

\* Functions & their inverses are reflections across  $y=x$  or  $f(x)=x$ .

$$6. \quad \sqrt[3]{352} = \sqrt[3]{7x^2 + 9}$$

$$\begin{array}{r} 352 = 7x^2 + 9 \\ -9 \qquad \qquad -9 \\ \hline \end{array}$$

$$\begin{array}{r} 343 = 7x^2 \\ \hline 7 \qquad \qquad 7 \\ \hline \sqrt{49} = \sqrt{x^2} \end{array}$$

$$\pm 7 = x$$

The image shows a screenshot of the Adobe Acrobat Reader DC interface. The window title is "sec3\_mod1\_funinv\_se\_31213 (17).pdf - Adobe Acrobat Reader DC". The menu bar includes "File", "Edit", "View", "Window", and "Help". The toolbar shows various navigation and editing tools. The main content area displays handwritten mathematical work in red ink. The first problem is labeled "5." and shows the equation  $(-4)^3 = (\sqrt[3]{5x+1})^3$ . A red checkmark is placed over the cube root symbol, and a red "3" is written above the right side of the equation. Below this, the equation  $-64 = 5x + 1$  is written, followed by three dots "..." indicating further steps. The second problem is labeled "7." and shows the equation  $5^x = 3125$ . The bottom status bar indicates a page size of "8.50 x 11.00 in".

5.  $(-4)^3 = (\sqrt[3]{5x+1})^3$   
 $-64 = 5x + 1$   
...

7.  $5^x = 3125$

The image is a screenshot of the Adobe Acrobat Reader DC interface. The window title is "sec3\_mod1\_funinv\_se\_31213 (17).pdf - Adobe Acrobat Reader DC". The menu bar includes "File", "Edit", "View", "Window", and "Help". The toolbar shows various navigation and editing tools, with the page number "14 / 37" and a zoom level of "300%".

In the center of the page, there is a handwritten math problem in blue ink:

$$11. \quad \cancel{3} \cdot 2^x = \frac{96}{\cancel{3}}$$
$$2^x = 32$$

Below the equations, there are three dots "...".

At the bottom of the page, there is a logo for "Mathematics Vision Project | MVP" and the text "Commercial-ShareAlike 3.0 Unported license". To the right of the text is a logo consisting of a red square and a gray square arranged in a 2x2 grid.

The status bar at the bottom left shows "8.50 x 11.00 in".



sec3\_mod1\_funinv\_se\_31213 (17).pdf - Adobe Acrobat Reader DC

File Edit View Window Help

Home Tools sec3\_mod1\_funinv\_... x

14 / 37 300%

2.  $2x^2 - 5 = 3x^2 - 12x + 31$   
 $\frac{-2x^2 + 5 \quad -2x^2 \quad + 5}{0 = x^2 - 12x + 36}$   
 $0 = (x - 6)(x - 6)$   
 $x = 6$

4.  $\sqrt{2x^2 + x - 2} = 2$   
 $2x^2 + x - 2 = 4$   
 $\frac{-4 \quad -4}{2x^2 + x - 6 = 0}$   
 $(2x^2 + 4x)(-3x - 6) = 0$   
 $2(x^2 + \frac{1}{2}x - 3) = 0$

~~$\begin{matrix} 2 & \cdot & -6 \\ a \cdot c & & -12 \\ 4 & & -3 \end{matrix}$~~   
 $\begin{matrix} 1 \\ 6 \end{matrix}$

~~$\begin{matrix} a \cdot c \\ 1 \cdot 36 \\ 36 \\ 6 \\ b \end{matrix}$~~

~~$\begin{matrix} 2 & \cdot & -6 \\ a \cdot c & & -12 \\ 4 & & -3 \end{matrix}$~~   
 $\begin{matrix} 1 \\ 6 \end{matrix}$

$(2x)(x+2)(-3)(x+2) = 0$

$(x+2)(2x-3) = 0$

$x = -2, \frac{3}{2}$

$x+2=0$

$2x-3=0$   
 $2x=3$   
 $x=\frac{3}{2}$

$$f(x) = 3x$$

$$g(x) = 10x + 4$$

$$h(x) = x^2 - x$$

Calculate the indicated function values.

15.  $f(7)$

16.  $f(-9)$

17.  $f(s)$

18.  $f(s - t)$

19.  $g(7)$

20.  $g(-9)$

21.  $g(s)$

22.  $g(s - t)$

24.  $h(7)$

25.  $h(-9)$

26.  $h(s)$

27.  $h(s - t)$

Notice that the notation  $f(g(x))$  is indicating that you replace  $x$  in  $f(x)$  with  $g(x)$ .

Simplify the following.

28.  $f(g(x)) =$

$$f(10x + 4) = 3(10x + 4) = 30x + 12$$

29.  $f(h(x)) =$

$$f(x^2 - x) = 3(x^2 - x) = 3x^2 - 3x$$

30.  $g(f(x)) =$

$$g(3x) = 10(3x + 4) =$$

$$30x + 40$$

Questions on 1.2 HW?

## QUIZ #2: Quadratic Functions & Their Inverses

For  $p(t) = 0.02t^2$  find the following:

1)  $p(20) =$

2)  $t$  when  $p(t) = 32$