Questions on Review #6?

Sep 19-9:00 AM

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13. The approximate *average* rate of change of the function $f(x)=\int_0^x \sin(t^2)\,dt$ over the interval $[1, 3]$ is

(A) 0.19 (B) 0.23 (C) 0.27 (D) 0.31 (E) 0.35

$$\frac{1}{2}\int_1^3 \sin x^2\,dx = \frac{0.463}{2} = \underline{0.2316}$$

14. Consider the function f defined on the domain $-0.5 \le x \le 0.5$ with $f(0)=1$, and $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \sec^2(3x)$. Evaluate: $\int_0^{0.5} f(x)\,dx$.

(A) 0.294
(B) 0.794
(C) 1.294
(D) 1.794
(E) 4.700

15. If $g(x)=\int_0^{x^2}(t^2+7)^{2/3}\,dt$, then $g''(1)=$

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17. If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

$$\int_1^4 \frac{x^2}{1+x^5} = F(4) - F(1) = F(4) - 0 = 0.3756$$

FREE RESPONSE – Calculator allowed

18. Consider the following table of values for the differentiable function f.

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	5.0	3.5	2.6	2.0	1.5

(a) Estimate $f'(1.4)$.

(b) Give an equation for the tangent line to the graph of f at $x = 1.4$.

(c) What is the sign of $f''(1.4)$? Explain your answer.

(d) Using the data in the table, find a midpoint approximation with 2 equal subdivisions for

$$\int_{1.0}^{1.8} f(x)\,dx.$$

Apr 24-9:33 AM

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a) $\dfrac{f(1.6)-f(1.2)}{1.6-1.2} = -3.75$

b) $y-2.6=-3.75(x-1.4)$

c) concave ↑ (+)

d) $\int_{1.0}^{1.8} f(x)dx = (3.5(0.4)+2.0(0.4))$

$= 2.2$

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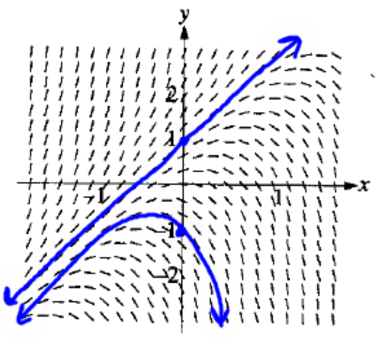
8. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.



(b) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

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(b) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

$$\frac{dy}{dx} = 2y - 4x$$

$$\frac{dy}{dx} = 2(2x+b) - 4x$$

$$\frac{dy}{dx} = \cancel{4x} + 2b - \cancel{4x}$$

$$\frac{dy}{dx} = 2b \qquad \frac{dy}{dx} = 2$$

$$2b = 2$$

$$b = 1$$

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer. $(0,0)$

$$g'(0) = \left.\frac{dy}{dx}\right|_{(0,0)} \qquad 2(0) - 4(0) = 0$$

$$g''(x) = 2 \cdot y' - 4, \text{ so}$$

$$g''(0) = 2 \cdot g'(0) - 4 = -4$$

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$-4 < 0$, so g has a local max at $(0,0)$.

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From Review #6

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

$$\int \frac{dy}{y^2} = \int (2x+2)dx$$

$y^{-2}dy$

$\frac{y^{-1}}{-1} \rightarrow -\frac{1}{y} = \frac{2x^2}{2} + 2x + C$

$$-\frac{1}{y} = x^2 + 2x + C$$

$(0,-1)$ $-\frac{1}{-1} = 0^2 + 2 \cdot 0 + C$

$\underline{1 = C}$

$$\cancel{y} \cdot -\frac{1}{\cancel{y}} = \frac{(x^2 + 2x + 1)y}{x^2 + 2x + 1}$$

$$\boxed{\frac{-1}{x^2+2x+1} = y \quad \text{or} \quad \frac{-1}{(x+1)^2} = y}$$

Applications: Population equations: (Logistic Growth Formula)

The solution of the general logistic differential equation $\frac{dP}{dt} = kP(m-P)$ is $P = \frac{m}{1+Ae^{-(mk)t}}$

where A is a constant determined by an appropriate initial condition. The carrying capacity m and the growth constant k are positive constants.

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right) \rightarrow \frac{dP}{dt} = \frac{1}{5}P(12-P)$$

so $k = \frac{1}{5}$

$m = 12$

a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$? If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 3$, $\lim_{t\to\infty} P(t) = 12$ and If $P(0) = 20$, $\lim_{t\to\infty} P(t) = 12$

b) If $P(0) = 3$, for what value of P is the population growing the fastest?

Population is growing the fastest when P is half the carrying capacity, $12 \div 2 \Rightarrow P = 6$.

Apr 7-2:43 PM

9.2 L'Hôpital's Rule

Indeterminate Form 0/0

If functions $f(x)$ and $g(x)$ are both zero at $x = a$, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ cannot be found by substituting $x = a$. The substitution produces 0/0, a meaningless expression known as an **indeterminate form**.

Other indeterminate forms: $\pm \frac{\infty}{\infty}$

L'Hopital's Rule (First Form)

Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$.

Sep 22-8:38 AM

Example: Indeterminate Form

Use L'Hôpital's Rule to find the limit.

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$$

$\frac{0}{0}$, so

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x} = \frac{\frac{1}{2}(4+x)^{-1/2}\cdot 1}{1} = \frac{\frac{1}{2}\cdot\frac{1}{\sqrt{4}}}{1} = \frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{1}{4}}$$

Let $f(x)=\sqrt{4+x}-2$ and $g(x)=x$. Since $f(0)=g(0)=0$ we can apply l'Hôpital's Rule where $f'(x)=\frac{1}{2}(4+x)^{-\frac{1}{2}}$ and $g'(x)=1$.

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}=\lim_{x\to 0}\frac{\frac{1}{2}(4+x)^{-\frac{1}{2}}}{1}=\frac{1}{4}$$

Example: One-Sided Limits

Evaluate using l'Hôpital's Rule: $\lim_{x \to 0^-} \frac{\sin x}{2x}$, $\frac{0}{0}$, so

$\lim_{x \to 0^-} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

Substituting $x = 0$ leads to the indeterminate form 0/0. Apply l'Hôpital's Rule: $\lim_{x \to 0^-} \frac{\sin x}{2x} = \lim_{x \to 0^-} \frac{\cos x}{2} = \frac{1}{2}$

Example: Indeterminate Form ∞/∞

Identify the indeterminate form and evaluate the limit using l'Hôpital's Rule. $\lim\limits_{x\to\pi} \dfrac{\csc x}{1+\cot x}$

The numerator and denominator are discontinuous at $x=\pi$, so we investigate the one-sided limits there. To apply l'Hôpital's Rule we can choose I to be any open interval containing $x=\pi$.

$$\lim_{x\to\pi^-} \frac{\csc x}{1+\cot x} = \frac{\infty}{\infty}$$

Next differentiate the numerator and denominator.

$$\lim_{x\to\pi^-} \frac{\csc x}{1+\cot x} = \lim_{x\to\pi^-} \frac{-\csc x \cot x}{-\csc^2 x} = \lim_{x\to\pi^-} \cos x = -1$$

The right hand limit is also equal to -1. Therefore, the two-sided limit is equal to -1.

Homework

9.2 pg.454-455 #3-21(X3), 33-52(X3)

Find out which form, solve only those that are $\frac{0}{0}$, $\pm\frac{\infty}{\infty}$