Graphing Rational Functions

Step 1 – Is your function fully simplified?

> If your function is in standard form, $f(x) = \frac{a}{x-h} + k$, do not worry about a getting a common denominator, just see if anything else can simplify away.

> Are your numerator and denominator both factored down completely? If not, factor both down and see if any of your factors simplify away.

Step 2 – What values of x will make the denominator equal to 0? These x-values will be your **vertical asymptotes**. If any of those factors simplified away in step 1, those x-values will be **holes** in your graph, not vertical asymptotes.

Step 3 – Is the function improper, with the degrees of the numerator and denominator equal to one another or with the numerator degree being bigger than denominator degree, or is it proper?

> This will help you to know what your **end behavior** is and also whether you have a **horizontal** or **slant asymptote**; you will only have one or the other, not both!

> **~Proper**: Your end behavior will approach 0 and you have a **horizontal** asymptote at y = 0 (the x-axis).
> **~Improper (n=d)**: Your end behavior will approach the ratio of the leading coefficients of the numerator and the denominator and you have a **horizontal** asymptote at that ratio.
> **~Improper (n>d)**: Your end behavior will approach +/- infinity and you have a **slant** asymptote, which you must find through long division – YAY! Your slant asymptote is only the quotient, not the remainder or the divisor.

Step 4 – What are your x- and y-intercepts?

> Mark these on your graph. To find your x-intercepts, set y=0 and solve for x; to find your y-intercepts, set x=0 and solve for y. If your rational function is **not** in standard form, $f(x) = \frac{a}{x-h} + k$, then your x-intercepts will be what makes the **numerator** equal to 0.

Step 5 – Complete a **sign line**; I like to include both my x-intercepts and also my vertical asymptotes/holes; especially if I am graphing without a calculator.

> A sign line is just a number line where you mark those x-intercepts and vertical asymptotes/holes, and you pick test values to plug into the original equation to see if at those test values, your y-values are positive or negative.

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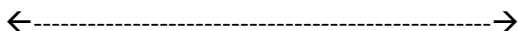
When you have used enough test points, if you're allowed to check with a calculator, check with a calculator; if not, turn it in and hope for the best!