Name: ____

AP CALCULUS AB
Differentiation Review

Fifteen derivatives you must know (*u* and *v* are functions of *x*; *a* is a number)

$f(x) = u^a$	$f'(x) = au^{a-1} \cdot u'$
$f(x) = e^u$	$f'(x) = e^u \cdot u'$
$f(x) = a^u$	$f'(x) = a^u \cdot \ln a \cdot u'$
$f(x) = \ln u$	$f'(x) = \frac{1}{u} \cdot u' = \frac{u'}{u}$
$f(x) = \log_a u$	$f'(x) = \frac{1}{u \cdot \ln a} \cdot u' = \frac{u'}{u \cdot \ln a}$
$f(x) = \sin u$	$f'(x) = \cos u \cdot u'$
$f(x) = \cos u$	$f'(x) = -\sin u \cdot u'$
$f(x) = \tan u$	$f'(x) = \sec^2 u \cdot u'$
$f(x) = \cot u$	$f'(x) = -\csc^2 u \cdot u'$
$f(x) = \sec u$	$f'(x) = \sec u \cdot \tan u \cdot u'$
$f(x) = \csc u$	$f'(x) = -\csc u \cdot \cot u \cdot u'$
$f(x) = \arcsin u$	$f'(x) = \frac{1}{\sqrt{1-u^2}} \cdot u' = \frac{u'}{\sqrt{1-u^2}}$
$f(x) = \arccos u$	$f'(x) = -\frac{1}{\sqrt{1-u^2}} \cdot u' = -\frac{u'}{\sqrt{1-u^2}}$
$f(x) = \arctan u$	$f'(x) = \frac{1}{1+u^2} \cdot u' = \frac{u'}{1+u^2}$
$f(x) = \operatorname{arc}\cot u$	$f'(x) = -\frac{1}{1+u^2} \cdot u' = -\frac{u'}{1+u^2}$

Types of differentiation (*u* and *v* are functions of *x*)

- Sum and Difference Rule:
  If $y = u \pm v$, then $y' = u' \pm v'$

Differentiate. Don't leave any fraction and/or negative exponents.

$$s(t) = 2t^3 - \pi t^2 + \sqrt{t} - \frac{1}{t} + 3$$

- Product Rule:
  If $y = u \cdot v$, then $y' = uv' + vu'$

  Differentiate. Don't leave any fraction and/or negative exponents.

$$f(x) = \frac{3}{x^2} \tan x$$

- Quotient Rule:
  If $y = \frac{u}{v}$, then $y' = \frac{vu' - uv'}{v^2}$

  Differentiate. Don't leave any fraction and/or negative exponents.

$$y = \frac{\sqrt{x} + 4}{3x^2}$$

- Chain Rule:
  If $y = u(v(x))$, then $y' = u'(v(x)) \cdot v'(x)$

  Differentiate.

$$g(x) = \ln(\cos(5 - 2x))$$

REMEMBER THAT the Chain Rule may need to be used in conjunction with the other rules for differentiation.

Differentiate and simplify.

$y = \sin^3 x \tan 4x$

$y = \frac{x}{\sqrt{1 + x^2}}$

- Implicit differentiation:

Use when the equation is difficult to solve for y.

Differentiate.

$$y^2 - 3x\sin y + x = 7$$

- Logarithmic differentiation:

Use when $y = u^v$

EXAMPLE:

$y = (\sin x)^{x^2}$

$\ln y = \ln\left[(\sin x)^{x^2}\right]$ — Take the natural log of both sides

$\ln y = x^2 \ln(\sin x)$ — Use the exponent property of logs

$\dfrac{d}{dx}\left(\ln y = x^2 \ln(\sin x)\right)$ — Take the derivative of both sides

$\dfrac{1}{y}\dfrac{dy}{dx} = x^2 \cdot \dfrac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 2x$ — Differentiate implicitly using appropriate rules

$\dfrac{1}{y}\dfrac{dy}{dx} = x^2 \cot x + 2x\ln(\sin x)$ — Simplify the right side

$\dfrac{dy}{dx} = y\left(x^2 \cot x + 2x\ln(\sin x)\right)$ — Solve for $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = (\sin x)^{x^2}\left(x^2 \cot x + 2x\ln(\sin x)\right)$ — Rewrite y as a function of x

Try one on your own. Differentiate $y = x^{\ln x}$.

Often, when you need to find a derivative, you need to use more than one of the above techniques. A few things that will make differentiation easier are memorizing the 15 basic derivatives you are supposed to know, changing all roots in a function to fraction exponents, changing variables in the denominator of a fraction to negative exponents, and remembering basic logarithmic identities.

For the following functions, state the type(s) of differentiation you will use. *Do not find the derivative.*

1. $f(x) = 3x^5 - 4x^3 - 3x$ ____________________

2. $y = \sqrt{16 - x}$ ____________________

3. $y = xy + x^2 + 1$ ____________________

4. $f(x) = xe^{\ln x^2}$ ____________________

5. $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$ ____________________

6. $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$ ____________________

7. $f(x) = \cos(2x) + \ln(3x)$ ____________________

8. $f(x) = x^3 + 3x^2 - 9x + 7$ ____________________

9. $y^2 + (xy+1)^3 = 0$ ____________________

10. $f(x) = 3x^5 - 5x^4$ ____________________

11. $y = (\sin x)^{e^x}$ ____________________

12. $f(x) = x^4 + 2x^2$ ____________________

13. $f(x) = 2xe^{2x}$ ____________________

14. $F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right)$ ____________________

On a separate piece of paper, differentiate functions 2, 4, 5, 7, 9, and 11.