Name: _____ Period: _____ Date: _____

AP CALCULUS AB
Unit 7 Review
Differential Equations and Mathematical Modeling

No calculator may be used on the following problems.

1. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx =$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

2. $\int x\sqrt{5x^2-4}\;dx =$

(A) $\frac{1}{10}(5x^2-4)^{\frac{3}{2}}+C$

(B) $\frac{1}{15}(5x^2-4)^{\frac{3}{2}}+C$

(C) $-\frac{1}{5}(5x^2-4)^{-\frac{1}{2}}+C$

(D) $\frac{20}{3}(5x^2-4)^{\frac{3}{2}}+C$

(E) $\frac{3}{20}(5x^2-4)^{\frac{3}{2}}+C$

3. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

4. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18 (B) $\sqrt{66}$ (C) 58 (D) $\sqrt{74}$ (E) $\sqrt{58}$

5. $\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\frac{x}{3}\right)+C$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

(C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right)+C$

(D) $\frac{1}{3}\tan^{-1}(x)+C$

(E) $\frac{1}{9}\tan^{-1}(x)+C$

6. $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $-\frac{2\pi}{3}$

7. $\int x\sqrt{x+3}\,dx =$

(A) $\frac{2}{3}(x)^{\frac{3}{2}} + 6(x)^{\frac{1}{2}} + C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3} + C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2} + C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3} + C$

A graphing calculator may be used on the following problems.

8. $\int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{-\frac{\pi}{4}}^{0} \cos x\,dx =$

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

9. $\int \tan^6 x \sec^2 x\,dx =$

(A) $\frac{\tan^7 x}{7} + C$

(B) $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$

(C) $\frac{\tan^7 x \sec^3 x}{21} + C$

(D) $7\tan^7 x + C$

(E) $\frac{2}{7}\tan^7 x \sec x + C$

10. $\int \frac{\ln x}{3x}\,dx =$

(A) $6\ln^2|x| + C$

(B) $\frac{1}{6}\ln(\ln|x|) + C$

(C) $\frac{1}{3}\ln^2|x| + C$

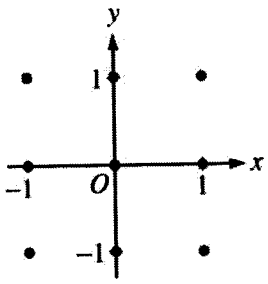
(D) $\frac{1}{6}\ln^2|x| + C$

(E) $\frac{1}{3}\ln|x| + C$

No calculator may be used on the following problem.

11. Consider the differential equation $\frac{dy}{dx} = (y-1)^2\cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.