Name: _____ Period: _____ Date: _____

AP CALCULUS AB
Unit 6 Review
Definite Integrals

No calculator may be used on the following problems.

1. The average value of $f(x)=\frac{1}{x}$ from $x = 1$ to $x = e$ is

(A) $\frac{1}{e+1}$

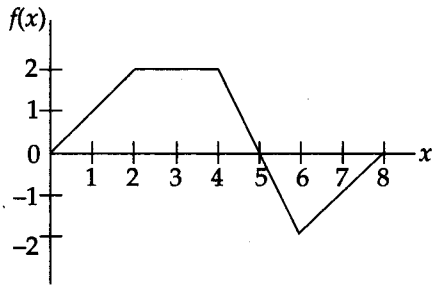
(B) $\frac{1}{1-e}$

(C) $e-1$

(D) $1-\frac{1}{e^2}$

(E) $\frac{1}{e-1}$

2.



The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

3. $\int_2^3 \frac{1}{x^3}\,dx =$

(A) $-\frac{5}{72}$ (B) $-\frac{5}{36}$ (C) $\frac{5}{144}$ (D) $\frac{5}{72}$ (E) $\ln\frac{27}{8}$

4. **If f is continuous for $a \le x \le b$, then at any point $x = c$, where $a < c < b$, which of the following must be true?**

(A) $f(c) = \dfrac{f(b) - f(a)}{b - a}$

(B) $f(a) = f(b)$

(C) $f(c) = 0$

(D) $\displaystyle\int_a^b f(x)\,dx = f(c)$

(E) $\lim\limits_{x \to c} f(x) = f(c)$

5. The average value of $\sec^2 x$ on the interval $\left[\dfrac{\pi}{6}, \dfrac{\pi}{4}\right]$ is

(A) $\dfrac{8}{\pi}$

(B) $\dfrac{12\sqrt{3} - 12}{\pi}$

(C) $\dfrac{12 - 4\sqrt{3}}{\pi}$

(D) $\dfrac{6\sqrt{2} - 6}{\pi}$

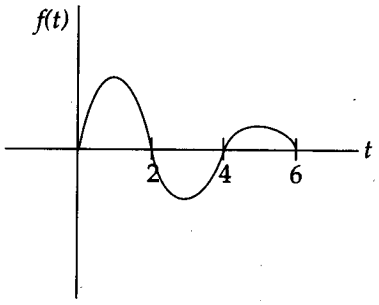
(E) $\dfrac{6 - 6\sqrt{2}}{\pi}$

A graphing calculator may be used for the following problems.

6. What is the trapezoidal approximation of $\displaystyle\int_0^3 e^x dx$ using $n = 4$ subintervals?

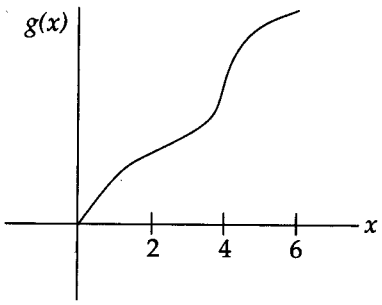
(A) 6.407 (B) 13.565 (C) 19.972 (D) 27.879 (E) 34.944

7.

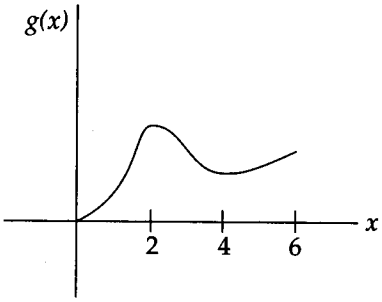


Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ has the graph shown above. Which of the following could be the graph of g?

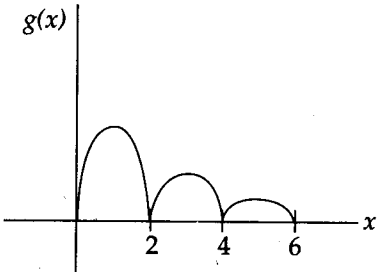
(A)



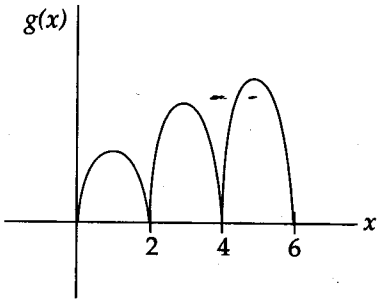
(B)



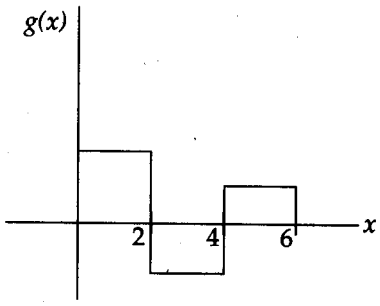
(C)



(D)



(E)



8. $\frac{d}{dx}\int_0^{x^2} \sin^2 t\, dt =$

(A) $x^2 \sin^2(x^2)$

(B) $2x\sin^2(x^2)$

(C) $\sin^2(x^2)$

(D) $x^2\cos^2(x^2)$

(E) $2x\cos^2(x^2)$

9. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

(A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm\frac{2}{3}$ (E) $\pm\frac{3}{2}$

10. Approximate $\int_0^1 \sin^2 x\, dx$ using the Trapezoid Rule with $n = 4$, to three decimal places.

(A) 0.277

(B) 0.273

(C) 0.555

(D) 1.109

(E) 2.219

11.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.