



AP CALCULUS SUMMER PACKET Highland High School

Welcome to AP Calculus! Get ready for a rigorous, challenging, and rewarding ride!

First, a little about the calculator that is strongly recommended for next year. As a school, we are transitioning to using the TI-nSpire calculator from 9th grade through 12th grade. If you are getting a calculator for this class, it should be a TI-nSpire CX. Do not get one that has a CAS as part of its name – those are not allowed on the AP test. There will be a few TI-nSpire available for checkout, but it will be best if you have your own. If you have a TI-84+ calculator, you may use it, but all the calculator instruction you will receive will be based on the TI-nSpire, so you will have to learn the TI-84 keystrokes on your own.

There are certain math skills you have been taught in the past that are essential for your success in AP Calculus. If you have not mastered these skills, you will find that your journey next year will be more difficult than it should be. It has been our experience that AP students are tripped up most often not because they lack calculus skills, but because they lack algebra and pre-calculus skills. This summer packet is intended to help you retain/review/relearn important concepts that you will use next year.

Please work all of the problems in the packet on separate pieces of paper, and show your work on each problem. We believe that you will benefit most from the packet if you start it the first or second week in July and work a few problems each day, as if it were a daily journal. Do not work the entire packet now (*yeah right, like THAT'S gonna happen ☺*), and do not wait until a week before you start school in August to cram it in. You are more likely to retain the information if you spread it out. This packet is due the first day of school and yes, it will be worth points toward your first term grade.

We have included at the bottom of this page a list of helpful websites for you to access if you get stuck. Feel free to use all internet and print resources available to you as you work through the packet. One great strategy that can optimize your chances for success this summer and next year is to connect with a couple of other students who will be taking AP Calculus next year and form a study/help group.

We're looking forward to seeing you in August. Have a great summer!

Ms. Welch
Ms. Hansen

Websites:

www.purplemath.com

www.mathtv.com/

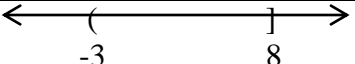
archives.math.utk.edu/visual/calculus/

www.khanacademy.org

Simplify, using only positive exponents.

1. -3^{-x}
2. $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$
3. $(16x^2y)^{\frac{3}{4}}$
4. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$
5. $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Interval notation. Complete the table.

Algebraic	Interval	Graph
$-1 \leq x$		
	$(3, 5]$	
		

Find the domain of the following functions. Make sure to use interval notation (Ex: $[0, 3)$)

6. $y = \log(2x-12)$
7. $y = \frac{x^2-4}{2x+4}$
8. $y = \frac{x^2-5x-6}{x^2-3x-18}$
9. $y = \frac{2^{2-x}}{x}$
10. $y = \sqrt{x-3} - \sqrt{x+3}$
11. $y = \frac{\sqrt{2x-9}}{2x+9}$

Factor completely.

12. $x^4 - 16$
13. $12x^3 - 6x^2 + 3x$
14. $x^5 + 11x^3 - 80x$

Describe in words the transformation(s) that would take place to $f(x)$ in each of the following.

15. $f(x) - 4$
16. $f(x-4)$
17. $-f(x+2)$
18. $5f(x) + 3$
19. $f(2x)$
20. $|f(x)|$

Determine algebraically if each function is even, odd, or neither. Show all work.

21. $f(x) = 2x^2 - 7$

22. $f(x) = -4x^3 - 2x$

23. $f(x) = 4x^2 - 4x + 4$

24. $f(x) = x - \frac{1}{x}$

Solve each equation by factoring, graphing, or using the quadratic formula.

25. $7x^2 - 3x = 0$

26. $4x(x - 2) - 5x(x - 1) = 2$

27. $x^2 + 6x + 4 = 0$

28. $2x^2 - 3x + 3 = 0$

29. $2x^2 - (x + 2)(x - 3) = 12$

30. $x + \frac{1}{x} = \frac{13}{6}$

Find the equations of all vertical ($x = a$ number) and horizontal ($y = a$ number) asymptotes, if they exist.

31. $y = \frac{x}{x - 3}$

32. $y = \frac{x + 4}{x^2 - 1}$

33. $y = \frac{x + 4}{x^2 + 1}$

34. $y = \frac{x^2 - 9}{x^3 + 3x^2 - 18x}$

35. $y = \frac{2x^3}{x^3 - 1}$

36. $y = \frac{\sqrt{x}}{2x^2 - 10}$

Simplify the following.

37. $\frac{x}{x - \frac{1}{2}}$

38. $\frac{\frac{1}{x} + 4}{\frac{1}{x} - 2}$

39. $\frac{\frac{x^2 - y^2}{xy}}{\frac{x + y}{y}}$

40. $\frac{\frac{x}{1 - x} + \frac{1 + x}{x}}{\frac{1 - x}{x} + \frac{x}{1 + x}}$

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find the following.

41. $f(g(2))$

42. $g(f(2))$

43. $f(h(-1))$

Solve each equation.

44. $\frac{2}{3} - \frac{5}{6} = \frac{1}{x}$

45. $x + \frac{6}{x} = 5$

46. $\frac{x+1}{3} - \frac{x-1}{2} = 1$

47. $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2 - 25}$

48. $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

49. $\frac{x-5}{x+1} = \frac{3}{5}$

Solve each equation on the interval $[0, 2\pi]$. Give exact values (Ex: $\frac{\pi}{3}$).

50. $\sin x = \frac{1}{2}$

51. $\cos^2 x = \cos x$

52. $2\cos x + \sqrt{3} = 0$

53. $4\sin^2 x = 1$

Solve the following problems, which address a variety of concepts.

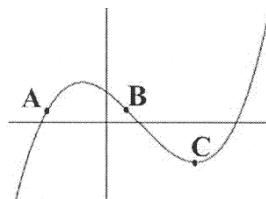
54. Let f be a linear function where $f(2) = -5$ and $f(-3) = 1$. Find $f(x)$.

55. Find an equation for the line, in point-slope form, that contains $(5, 1)$ and is perpendicular to $6x - 3y = 2$.

56. Use the table to calculate the average rate of change from $t = 1$ to $t = 4$.

t	0	1	2	3	4
$x(t)$	8	7	5	1	2

57. Order the points A, B, and C, from least to greatest **by their rates of change**.



58. Find the distance between the points $(8, -1)$ and $(-4, -6)$.

59. If $g(x) = \frac{x}{x+3}$ find $g^{-1}(x)$. (i.e., the inverse of g)

60. Find the points of intersection algebraically for the graphs of $y = x - 1$ and $y^2 = 2x + 6$.

61. Rewrite $\frac{1}{2} \ln(x-3) + \ln(x+2) - 6 \ln x$ as a single logarithmic expression.

62. Evaluate the following without a calculator. Give exact answers.

(a) $\sin \frac{7\pi}{6}$

(b) $\csc 60^\circ$

(c) $\cos 120^\circ$

(d) $\sec\left(-\frac{2\pi}{3}\right)$

(e) $\tan \frac{\pi}{2}$

(f) $\cot(-135^\circ)$

63. Sketch a graph of the piecewise function $f(x) = \begin{cases} x^2 - 5, & x < -1 \\ 0, & x = -1 \\ 6 - 4x, & x > -1 \end{cases}$

64. Describe the left- and right-end behaviors of the function $f(x) = -3^x$.

65. Find the domain and range of each function without a calculator.

(a) $f(x) = (x-3)^2 + 2$

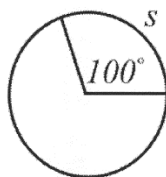
(b) $f(x) = 2|x-4| - 3$

(c) $f(x) = \sqrt[3]{1-x}$

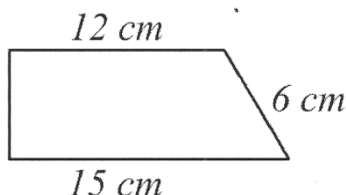
(d) $f(x) = 5 \sin(x)$

(e) $f(x) = e^{-x}$

66. The circle below has radius 6 ft. Find the area and circumference of the circle, then find s .



67. Find the area of the trapezoid.

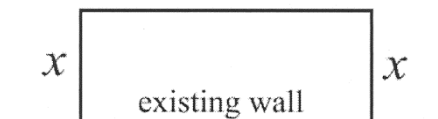


68. Find the volume of a washer with outer radius 18 ft., inner radius 15 ft., and height 3 ft.



69. Rewrite $\log_5(x+3)$ as an equivalent expression with only natural logarithms.

70. Three sides of a fence and an existing wall form a rectangular enclosure. The total length of fence used for the three sides is 240 feet. Find x if the area enclosed is 5500 ft^2 .



71. The number of elk after t years in a state park is modeled by the function $P(t) = \frac{1216}{1 + 75e^{-0.03t}}$

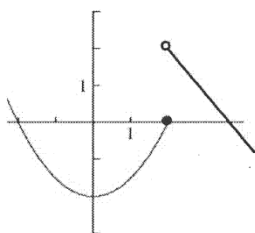
- (a) What is the initial population?
- (b) When will the number of elk be 750?
- (c) What is the maximum number of elk possible in the park?

72. Simplify $\csc x - \tan x \sin x \cos x$.

73. Use long division or synthetic division to simplify the expression $\frac{x^3 - 7x^2 + 14x - 8}{x - 4}$.

74. Use a graphing calculator to solve $e^{2x} = 3x^2$.

75. The function $f(x)$ is graphed below. Find the following.



a) $f(2)$

b) $f(0)$

c) $f(x) = 0$

76. Write the trig ratios for each of the following standard angles of the unit circle.

Angle (radians)	Cos	Tan	Sin	Csc	Sec	Cot
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{2\pi}{3}$						
$\frac{3\pi}{4}$						
$\frac{5\pi}{6}$						
π						
$\frac{7\pi}{6}$						
$\frac{5\pi}{4}$						
$\frac{4\pi}{3}$						
$\frac{3\pi}{2}$						
$\frac{5\pi}{3}$						
$\frac{7\pi}{4}$						
$\frac{11\pi}{6}$						
2π						