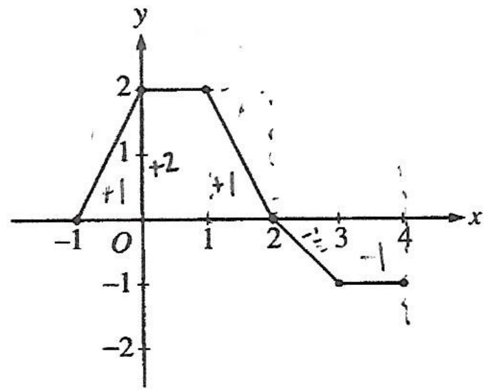
Name: Key Period: _____ Date: __________

AP CALCULUS AB
Unit 8 Review
Applications of Integrals

No calculator may be used on the following problems.

B 1.

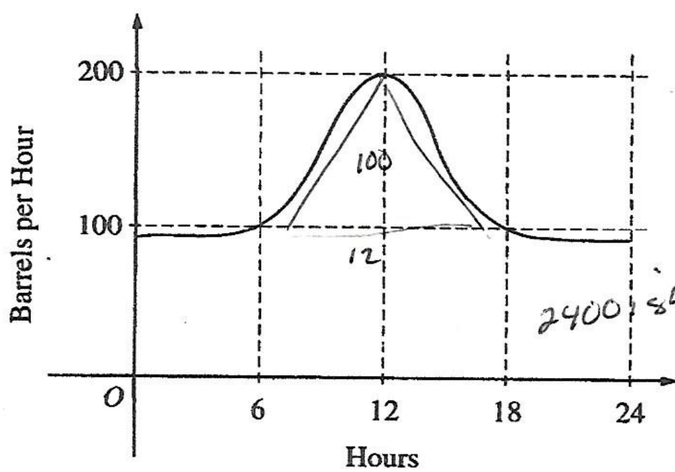


The graph of a piecewise-linear function f, for $-1 \leq x \leq 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

$4 - 1\frac{1}{2} = 2.5$

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

D 2.



$\frac{1}{2} \cdot 1200$

600

2400 ish

The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

C 3. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$v = 2t - 6$

$0 = 2t - 6$

$6 = 2t$

$3 = t$

D 4. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

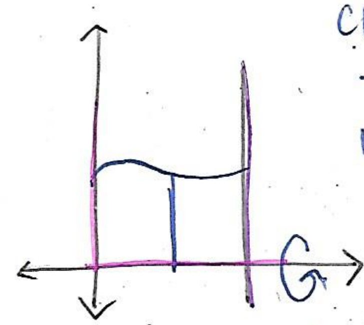
(A) $\pi\int_1^8 (x^2+1)^3\,dx$

(B) $\pi\int_1^8 (x^2+1)^6\,dx$

(C) $\pi\int_0^1 (x^2+1)^3\,dx$

(D) $\pi\int_0^1 (x^2+1)^6\,dx$

(E) $2\pi\int_0^1 (x^2+1)^6\,dx$



cross-sections - circles w/ radius $(x^2+1)^3$

$A(x) = \pi\left[(x^2+1)^3\right]^2$

$V = \int_0^1 \pi (x^2+1)^6$

C 5. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$?

(A) $\int_{-2}^{4}(x^2-2x-8)\,dx$

(B) $\int_{-4}^{2}(8+2x-x^2)\,dx$

(C) $\int_{-2}^{4}(8+2x-x^2)\,dx$

(D) $\int_{-4}^{2}(x^2-2x-8)\,dx$

(E) $\int_{2}^{4}(8+2x-x^2)\,dx$

$0 = -x^2 + 2x + 8$

$0 = -(x^2 - 2x - 8)$

$0 = -(x-4)(x+2)$

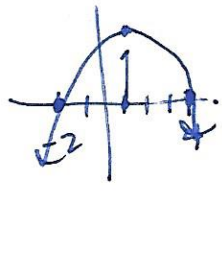
$x = 4, -2$

x-axis $\to y = 0$

upper area - lower area

$\int_{-2}^{4}\left[(8+2x-x^2) - 0\right]dx$

$\int_{-2}^{4}(8+2x-x^2)\,dx$



6. A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

D

$V = \pi\int_0^{\pi/2} (1+\sin^2 x)^2\,dx$

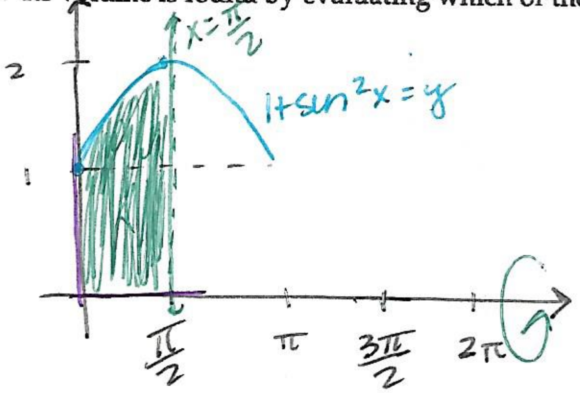
(A) $\pi\int_0^1 \left(1+\sin^4 x\right) dx$

(B) $\pi\int_0^1 \left(1+\sin^2 x\right)^2 dx$

(C) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^4 x\right) dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^2 x\right)^2 dx$

(E) $\pi\int_0^{\frac{\pi}{2}} \left(1+\sin^2 x\right) dx$



cross-sections
circles w/ radius
$r = 1+\sin^2 x$
$A(x) = \pi(1+\sin^2 x)^2$

A graphing calculator may be used on the following problems.

C 7. The volume generated by revolving about the y-axis the region enclosed by the graphs $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \le x \le 2$, is

SOLVE FOR X!

$\hookrightarrow x^2 = 9 - y$
$x = \sqrt{9-y}$
$x = \sqrt{-y+9}$

$\rightarrow 3x = 9 - y$
$x = \frac{9-y}{3}$

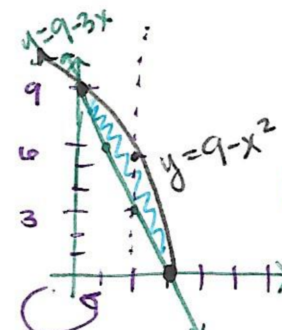
(A) -8π (B) 4π (C) 8π (D) 24π (E) 48π

washers →
outer circle:
$r \rightarrow x = \sqrt{9-y}$
$A(x) = \pi[\sqrt{9-y}]^2$
$= \pi(9-y)$
inner circle:
$r \rightarrow x = 3 - \frac{y}{3}$
$A(x) = \pi(3-\frac{y}{3})^2$

$V = \pi\int_0^9 \left[(9-y) - \left(3-\frac{y}{3}\right)^2\right] dy$

$V = 2\pi\int_0^2 x(\cancel{9} - x^2 - \cancel{9} + 3x)\,dx$

$V = 2\pi\int_0^2 (-x^3 + 3x^2)\,dx = 8\pi$



$(3-\frac{y}{3})(3-\frac{y}{3})$
$9 - y - y + \frac{y^2}{9}$
$9 - 2y + \frac{y^2}{9}$

B 8. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-t^2}$; where t stands for time.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

Dist. $= \int_0^4 v(t)\,dt = \int_0^4 7e^{-t^2}\,dt \approx 6.20359$

C 9. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

(A) 6.000

(B) 1.609

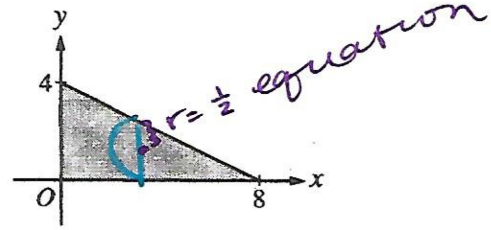
(C) 16.047

(D) 0.800

(E) 148.413

$\text{Dist} = \int_1^5 (t + \ln t)\,dt \approx 16.0472$

C 10.



The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

$\frac{2y}{2} = \frac{-x+8}{2}$

$y = -\frac{1}{2}x + 4$

semi-circle:

radius: $r = \frac{1}{2}\left(-\frac{x}{2} + 4\right)$

$= -\frac{x}{4} + 2$ or $2 - \frac{x}{4}$

$A(x) = \frac{1}{2}\pi r^2$

$A(x) = \frac{\pi}{2}\left(2 - \frac{x}{4}\right)^2$

$V = \frac{\pi}{2}\int_0^8 \left(2 - \frac{x}{4}\right)^2 dx$

$= \frac{\pi}{2}\left(10\frac{2}{3}\right)$

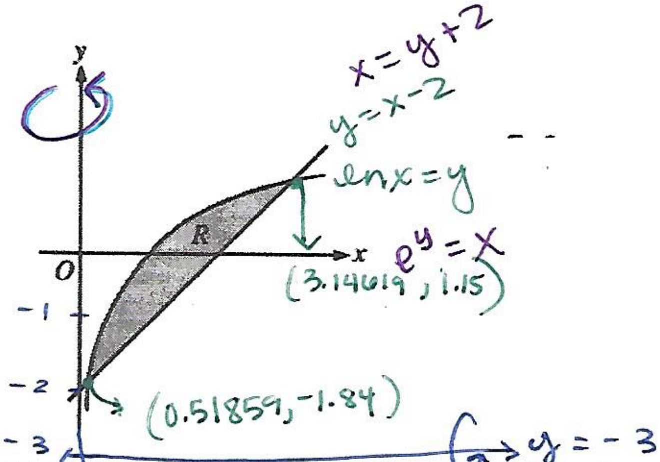
≈ 16.7552

11.

$\ln x = x - 2$

$x = 0.51859$ &

3.14619



Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

solve for x!

(a) Area $= \int_{0.51859}^{3.14619} (\ln x - (x-2))\,dx = \int_{0.51859}^{3.14619} (\ln x - x + 2)\,dx = 1.75948$

≈ 1.76 units^2

(b) washers/rings:

outer radius:

$r = \ln x - -3 = \ln x + 3$

$A(x) = \pi(\ln x + 3)^2$

inner radius:

$r = x - 2 - -3 = x - 2 + 3 = x + 1$

$A(x) = x + 1$

$V = \pi \int_{0.51859}^{3.14619} \left((\ln x + 3)^2 - (x+1)^2\right) dx = \pi(10.2428)$

≈ 32.1788

units^3

(c) $V = \pi \int_{-1.84}^{1.15} \left[(y+2)^2 - (e^y)^2\right] dy$