Name: hansen Period: _____ Date: ____________

AP CALCULUS AB
Unit 7 Review
Differential Equations and Mathematical Modeling

No calculator may be used on the following problems.

D 1. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx =$ $4\arctan x\Big]_{-1}^{1} = 4(\arctan 1 - \arctan -1) = 4\left(\frac{\pi}{4} - \frac{-\pi}{4}\right)$

$= 4\left(\frac{2\pi}{4}\right)$

$= 2\pi$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

B 2. $\frac{1}{10}\int x\sqrt{5x^2-4}\,dx = \frac{1}{10}\int \sqrt{u}\,du = \frac{1}{10}\int u^{1/2}\,du = \frac{1}{10}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{15}(5x^2-4)^{3/2} + C$

$u = 5x^2 - 4$

$\frac{du}{10} = \frac{10x\,dx}{10}$

$\frac{1}{10}du = x\,dx$

(A) $\frac{1}{10}(5x^2-4)^{\frac{3}{2}} + C$

(B) $\frac{1}{15}(5x^2-4)^{\frac{3}{2}} + C$

(C) $-\frac{1}{5}(5x^2-4)^{-\frac{1}{2}} + C$

(D) $\frac{20}{3}(5x^2-4)^{\frac{3}{2}} + C$

(E) $\frac{3}{20}(5x^2-4)^{\frac{3}{2}} + C$

B 3. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{5-1}\int_1^5 (x-1)^2\,dx = \frac{1}{4}\int u^2\,du = \left[\frac{1}{4}\cdot\frac{u^3}{3}\right]_1^5 = \left[\frac{u^3}{12}\right]_1^5 =$

$u = x-1$

$du = dx$

$= \frac{(x-1)^3}{12}\Big]_1^5 = \frac{4^3}{12} - 0 = \frac{64}{12}$

$= \frac{32}{6} = \frac{16}{3}$

$y^2 = 2x^3 + 4x - 8$

$y = \sqrt{2x^3+4x-8}$

$y = \sqrt{2\cdot 3^3 + 4\cdot 3 - 8}$

$y = \sqrt{54+12-8}$

$y = \sqrt{66-8}$

$y = \sqrt{58}$

E 4. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18 (B) $\sqrt{66}$ (C) 58 (D) $\sqrt{74}$ (E) $\sqrt{58}$

$\int y\,dy = \int (3x^2+2)dx$

$\frac{y^2}{2} = \frac{\cancel{3}x^3}{\cancel{3}} + 2x + C$

$\frac{y^2}{\cancel{2}} = x^3 + 2x + C$

$\frac{4^2}{2} = 2^3 + 2\cdot 2 + C$

$8 = 8 + 4 + C$

$-4 = C$

B 5. $\int \frac{dx}{9+x^2} = \frac{1}{9}\int \frac{dx}{1+\left(\frac{x}{3}\right)^2} = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

$\frac{9+x^2}{9}$

$1 + \frac{x^2}{9}$

$1 + \left(\frac{x}{3}\right)^2$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

(A) $3\tan^{-1}\left(\frac{x}{3}\right) + C$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

(C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right) + C$

(D) $\frac{1}{3}\tan^{-1}(x) + C$

(E) $\frac{1}{9}\tan^{-1}(x) + C$

B 6. $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx =$ $2\arcsin x \Big]_0^{1/2} = 2(\arcsin\frac{1}{2} - \arcsin 0)$ $= 2(\frac{\pi}{6} - 0) = \frac{\pi}{3}$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $-\frac{2\pi}{3}$

solve for x!

$u = x+3 \rightarrow x = u-3$

$du = dx$

$\int x(x+3)^{1/2}dx =$

$\int (u-3)(u-3+3)^{1/2}du =$

$\int (u-3)(u)^{1/2}du = \int (u^{3/2} - 3u^{1/2})\,du =$

$\frac{u^{5/2}}{5/2} - \frac{3u^{3/2}}{3/2} + C = \frac{2}{5}u^{5/2} - 2u^{3/2} + C =$

$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

C 7. $\int x\sqrt{x+3}\,dx =$

(A) $\frac{2}{3}(x)^{\frac{3}{2}} + 6(x)^{\frac{1}{2}} + C$

(B) $\frac{2(x+3)^{\frac{3}{2}}}{3} + C$

(C) $\frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$

(D) $\frac{3(x+3)^{\frac{3}{2}}}{2} + C$

(E) $\frac{4x^2(x+3)^{\frac{3}{2}}}{3} + C$

A graphing calculator may be used on the following problems.

D 8. $\int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{\frac{\pi}{4}}^{0} \cos x\,dx =$ 0.293 − 0.107 = 1 fnInt both

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

A 9. $\int \tan^6 x \underbrace{\sec^2 x\,dx}_{du} =$ $\int u^6 du = \frac{u^7}{7} + C = \frac{\tan^7 x}{7} + C$

$u = \tan x$

$du = \sec^2 x\,dx$

(A) $\frac{\tan^7 x}{7} + C$

(B) $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$

(C) $\frac{\tan^7 x \sec^3 x}{21} + C$

(D) $7\tan^7 x + C$

(E) $\frac{2}{7}\tan^7 x \sec x + C$

10. $\int \frac{\ln x}{3x}\,dx = \frac{1}{3}\int \frac{\ln x}{x}\,\overset{du}{dx} = \frac{1}{3}\int u\,du = \frac{1}{3}\cdot\frac{u^2}{2} + C = \frac{1}{6}u^2 + C$

$= \frac{1}{6}\ln^2|x| + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

(A) $6\ln^2|x| + C$

(B) $\frac{1}{6}\ln(\ln|x|) + C$

(C) $\frac{1}{3}\ln^2|x| + C$

(D) $\frac{1}{6}\ln^2|x| + C$

(E) $\frac{1}{3}\ln|x| + C$

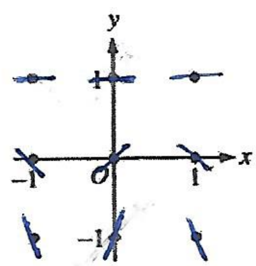
No calculator may be used on the following problem.

11.

Consider the differential equation $\frac{dy}{dx} = (y-1)^2\cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

$(-1-1)^2$
$4\cdot\cos -\pi = -4$
$4\cdot\cos 0 = 4$
$4\cdot\cos -\pi = -4$



$(-1)^2(\cos 0) = 1$
$(1-1)^2\cos\pi 0 = 0$
$(1-1)^2\cos -\pi = 0$
$(1-1)^2\cos\pi = -$
$1\cdot\cos\pi = -1$

(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

b) $y = 1$, so $c = 1$

c) $\frac{dy}{dx} = (y-1)^2\cos(\pi x)$

$\int \frac{dy}{(y-1)^2} = \int \cos(\pi x)\,dx$ $\quad u = \pi x$, $du = \pi dx$, $\frac{du}{\pi} = \frac{\pi}{\pi}$, $\frac{1}{\pi}du = dx$

$u = y-1$, $du = dy$: $\int \frac{du}{u^2} = \frac{1}{\pi}\int \cos u\,du$

$\frac{u^{-1}}{-1} = \frac{1}{\pi}\sin u + C$

$-\frac{1}{y-1} = \frac{1}{\pi}\sin(\pi x) + C$

$(1, 0)$

$\frac{-1}{0-1} = \frac{1}{\pi}\sin(\pi\cdot 1) + C$

$-\frac{1}{-1} = \frac{\sin\pi}{\pi} + C$

$1 = 0 + C$

$1 = C$

$\frac{-1}{(y-1)} = \frac{1}{\pi}\sin(\pi x) + 1$

$\frac{-1}{(y-1)} = \frac{\sin(\pi x) + \pi}{\pi}$

$\frac{-\pi}{\sin(\pi x)+\pi} = \frac{(y-1)(\sin(\pi x)+\pi)}{\sin(\pi x)+\pi}$

$\frac{-\pi}{\sin(\pi x)+\pi} + 1 = y - \cancel{1} \cancel{+1}$

$\frac{-\pi}{\sin(\pi x)+\pi} + 1 = y$

for $-\infty < x < \infty$