Name: Hansen Period: _____ Date: ____________

AP CALCULUS AB
Unit 5 Review
Applications of the Derivative

E

No calculator may be used on this portion of the test.

1. The value of c which satisfies the Mean Value Theorem for derivatives on the interval [0, 5] for the function $f(x) = x^3 - 6x$ is

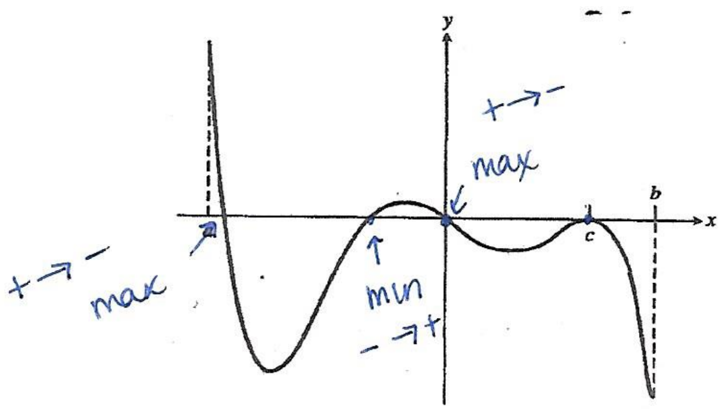
(A) $-\frac{5}{\sqrt{3}}$ (B) ~~0~~ (C) ~~1~~ (D) ~~$\frac{5}{3}$~~ (E) $\frac{5}{\sqrt{3}}$

$f(0) = 0 - 0 = 0$
$(0,0)$
$f(5) = 95$
$(5,95)$
$f'(x) = 3x^2 - 6$
$f'(c) = 3c^2 - 6$

$f'(c) = \frac{f(b) - f(a)}{(b-a)}$
$3c^2 - 6 = \frac{95-0}{5-0}$
$3c^2 - 6 = 19$
$3c^2 = 25$
$\sqrt{c^2} = \sqrt{\frac{25}{3}}$
$c = \frac{5}{\sqrt{3}}$

E

2. The graph of $f'(x)$ is shown. It is tangent to the x-axis at point c. Which of the following describes all relative extrema of $f(x)$ on the open interval (a, b)?



(A) One relative maximum and one relative minimum

(B) One relative maximum and two relative minima

(C) Three relative maxima and two relative minima

(D) Two relative maxima and two relative minima

(E) Two relative maxima and one relative minimum

C

3. The function f defined by $f(x) = x^4 - x^2$ has a relative minimum at $x =$

(A) $\sqrt{2}$ (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{1}{2}$ (E) 0

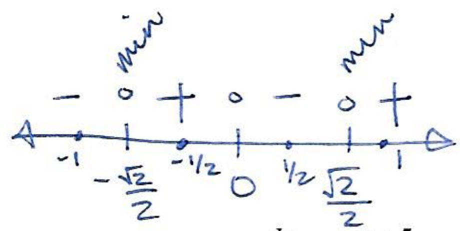
$f'(x) = 4x^3 - 2x$

$0 = 2x(2x^2 - 1)$

$0 = 2x(\sqrt{2}x - 1)(\sqrt{2}x + 1)$

$x = 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

f'



$f\left(-\frac{\sqrt{2}}{2}\right) =$

$f(0) =$

$f\left(\frac{\sqrt{2}}{2}\right) =$

E

4. Let y be a differentiable function such that $y(-2) = 7$ and $\frac{dy}{dx} = \frac{-x+5}{2y}$. A linear approximation of $y(-1.8)$ is

$(-2, 7)$

(A) -7.9 (B) -7.1 (C) -1.4 (D) 5.1 (E) 7.1

$\frac{-(-2)+5}{2(7)} =$

$\frac{7}{14} = \frac{1}{2}$

$L(x) = 7 + \frac{1}{2}(x+2)$

$L(-1.8) = \frac{1}{2}(-1.8) + 8$

$= 7.1$

$L(x) = 7 + \frac{1}{2}x + 1$

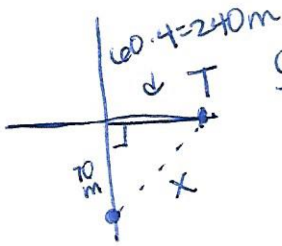
$L(x) = \frac{1}{2}x + 8$

A graphing calculator may be used on the following portion of the test.

A

5. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40



$\frac{dT}{dt} = 60$

$T = 240$

$70^2 + T^2 = x^2$

$2T\frac{dT}{dt} = 2x\frac{dx}{dt}$

$2(240)(60) = 2(250)\frac{dx}{dt}$

$\frac{dx}{dt} = 57.6$

$70^2 + 240^2 = x^2$

GC

D 6. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

$y' = 3x^2 + 12x + 7 + 2\sin x$

$y'' = 6x + 12 + 2\cos x$

graph $(-1.89, 0)$

GC

E 7. Find two non-negative numbers x and y whose sum is 100 and for which $x^2 y$ is a maximum.

$x > 0$

$y > 0$

(A) $x = 33.333$ and $y = 33.333$

(B) $x = 50$ and $y = 50$

(C) $x = 33.333$ and $y = 66.667$

(D) $x = 100$ and $y = 0$

(E) $x = 66.667$ and $y = 33.333$

$x + y = 100$

$y = 100 - x$

$x = 66.7$

$y = 100 - 66.7$

$y = 33.3$

$x^2 y = 0$

$x^2(100 - x) = 0$

$100x^2 - x^3 = A$

$\frac{dA}{dx} = 200x - 3x^2$

$0 = x(200 - 3x)$

$x = 0$

$200 - 3x = 0$

$\frac{200}{3} = \frac{3x}{3}$

$66.7 \approx 200/3 = x$

FREE RESPONSE. A graphing calculator may be used for these problems.

8. Consider the curve defined by $y = x^4 + 4x^3$.

(a) Find the equation of the tangent line to the curve at $x = -1$.

$y' = 4x^3 + 12x^2$

$y'(-1) = -4 + 12 = 8 = m$

$y(-1) = 1 - 4 = -3$

$(-1, -3)$

$y - y_1 = m(x - x_1)$

$y + 3 = 8(x + 1)$ or

$y = 8x + 8 - 3$

$y = 8x + 5$

(b) Find the coordinates of the absolute minimum.

$0 = 4x^3 + 12x^2$

$0 = 4x^2(x + 3)$

$x = 0, -3$

$f'(-4) = -$ 0 $f'(-1) = +$ 0 $f'(1) = +$

-4 -3 -1 0 1 y'

$- \to + =$ abs. min.

$f(-3) = -27$ * absolute min @ $(-3, -27)$

$y = x^4 + 4x^3$

$y' = 4x^3 + 12x^2$

where graph has a tan line and concavity changes

(c) Find the coordinates of the point(s) of inflection.

$y'' = 12x^2 + 24x$

$0 = 12x(x+2)$

$x = 0, -2$

$f''(-3) = +$ 0 $f''(-1) = -$ 0 $f''(1) = +$

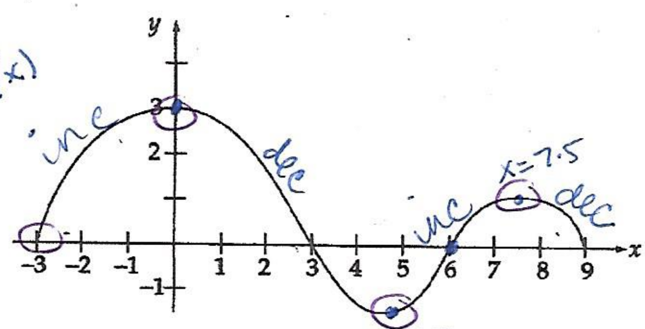
y'': −3, −2, −1, 0, 1

$f(-2) = 16$ $f(0) = 0$

★Points of inflection: $(-2, 16)$, $(0, 0)$

9.

$g(x) = f'(x)$



concave ↑ where y' is inc.

concave ↓ where y' is dec

The figure above shows the graph of $g(x)$, where g is the derivative of the function f, for $-3 \le x \le 9$. The graph consists of three semicircular regions and has horizontal tangent lines at $x = 0$, $x = 4.5$, and $x = 7.5$.

(a) Find all values of x, for $-3 < x \le 9$, at which f attains a relative minimum. Justify your answer.

f' − → +

$g(x) = 0$ @ $x = -3, 3, 6, 9$

★ $x = 6$ is a relative min because $g(x) = f'(x)$ changes sign from negative to positive

★$x = -3$ is also a relative min because $f'(x) = g(x) > 0$ for all $x > -3$.

(b) Find all values of x, for $-3 < x \le 9$, at which f attains a relative maximum. Justify your answer.

★ $x = 3$ is a relative max because $g(x) = f'(x)$ changes sign from positive to negative.

f' + → −

★ $x = 9$ is a relative max because $f'(x) = g(x) > 0$ for all $x < 9$.

(d) Find all points where $f''(x) = 0$.

$f''(x) = 0$ at $x = 0, 4.5, 7.5$ because the graph of $f'(x) = g(x)$ has a local max/min at these values.