Name: Hansen Date: ______

AP CALCULUS AB
Unit 6 Review
Definite Integrals

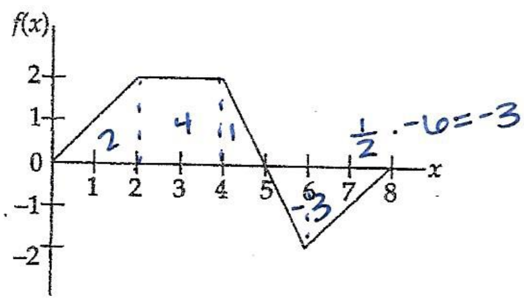
No calculator may be used on the following problems.

D 1. $\int_2^3 \frac{1}{x^3}\,dx =$

(A) $-\frac{5}{72}$ (B) $-\frac{5}{36}$ (C) $\frac{5}{144}$ (D) $\frac{5}{72}$ (E) $\ln\frac{27}{8}$

$\int_2^3 x^{-3}dx = \frac{x^{-2}}{-2}\Big]_2^3 = \frac{1}{-2x^2}\Big]_2^3 = \frac{1}{-18} + \frac{1}{+8} = \frac{-8+18}{144} = \frac{10}{144} = \frac{5}{72}$

B 2.



$2 + 4 + 1 - 3 = 4$

The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1 (B) 4 (C) 8 (D) 10 (E) 13

B 3. If $\int_a^b f(x)\,dx = 5$ and $\int_a^b g(x)\,dx = -1$, which of the following must be true?

F I. $f(x) > g(x)$ for $a \le x \le b$

T II. $\int_a^b \left[f(x) + g(x)\right]dx = 4$

F III. $\int_a^b \left[f(x)g(x)\right]dx = -5$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

E 4. If f is continuous for $a \le x \le b$, then at any point $x = c$, where $a < c < b$, which of the following must be true?

(A) $f(c) = \dfrac{f(b) - f(a)}{b - a}$

(B) $f(a) = f(b)$

(C) $f(c) = 0$

(D) $\int_a^b f(x)\,dx = f(c)$

(E) $\lim_{x \to c} f(x) = f(c)$

C 5. The average value of $\sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ is

(A) $\dfrac{8}{\pi}$

(B) $\dfrac{12\sqrt{3} - 12}{\pi}$

(C) $\dfrac{12 - 4\sqrt{3}}{\pi}$

(D) $\dfrac{6\sqrt{2} - 6}{\pi}$

(E) $\dfrac{6 - 6\sqrt{2}}{\pi}$

$$\frac{1}{\frac{3\pi}{4} - \frac{2\pi}{6}} \int_{\pi/6}^{\pi/4} \sec^2 x\,dx = \frac{12}{\pi}\Big[\tan x\Big]_{\pi/6}^{\pi/4} =$$

12

$$\frac{12}{\pi}\left[\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right] = \frac{12}{\pi}\left[1 - \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\right] =$$

$$\frac{12}{\pi}\left[\frac{3-\sqrt{3}}{3}\right] = \frac{\overset{4}{\cancel{12}}(3-\sqrt{3})}{\underset{1}{\cancel{3}}\pi} = \frac{12 - 4\sqrt{3}}{\pi}$$

D 6. $\int_{\frac{\pi}{2}}^{x} \cos t\,dt =$

(A) $-\sin x$ (B) $-\sin x - 1$ (C) $\sin x + 1$ (D) $\sin x - 1$ (E) $1 - \sin x$

$$\sin t\Big]_{\pi/2}^{x} = \sin x - \sin\frac{\pi}{2} = \sin x - 1$$

A graphing calculator may be used for the following problems.

B 7. The expression $\frac{1}{30}\left(\sin\frac{1}{30}+\sin\frac{2}{30}+\sin\frac{3}{30}+\ldots+\sin\frac{30}{30}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sin\frac{x}{30}\,dx$

(B) $\int_0^1 \sin x\,dx$

(C) $\frac{1}{30}\int_0^1 \sin\frac{x}{30}\,dx$

(D) $\frac{1}{30}\int_0^1 \sin x\,dx$

(E) $\frac{1}{30}\int_0^{30} \sin x\,dx$

B 8. $\frac{d}{dx}\int_0^{x^2}\sin^2 t\,dt =$

(A) $x^2\sin^2(x^2)$

(B) $2x\sin^2(x^2)$

(C) $\sin^2(x^2)$

(D) $x^2\cos^2(x^2)$

(E) $2x\cos^2(x^2)$

$2x\cdot\sin^2(x^2)$

C 9. A continuous function $h(t)$ is defined in the closed interval $[10, 16]$ with values given in the table below. Using the data, estimate $\int_{10}^{16} h(t)\,dt$ using a trapezoidal approximation with three subintervals of unequal length.

2 3 1

t	10	12	15	16
$h(t)$	10	20	50	80

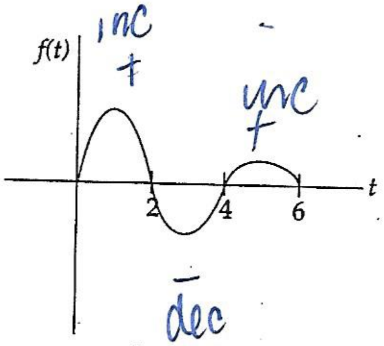
(A) $\frac{359}{3}$ (B) 130 (C) 200 (D) 270 (E) 718

$\frac{1}{2}\cdot 2(10+20)+\frac{1}{2}\cdot 3(20+50)+\frac{1}{2}\cdot 1(50+80) =$

$30+\frac{3}{2}\cdot\overset{35}{\cancel{70}}+65 =$

30
105
65
200

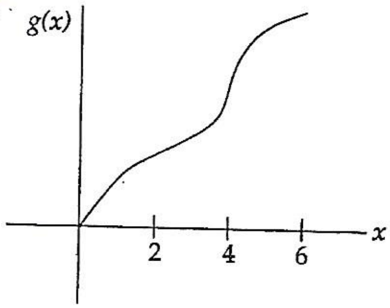
B 10.



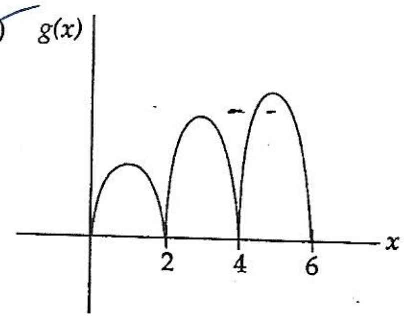
$g(x) = \int_0^x f(t)\,dt = F(x) - F(0) = F(x)$

Let $g(x)=\int_0^x f(t)\,dt$, where $f(t)$ has the graph shown above. Which of the following could be the graph of g?

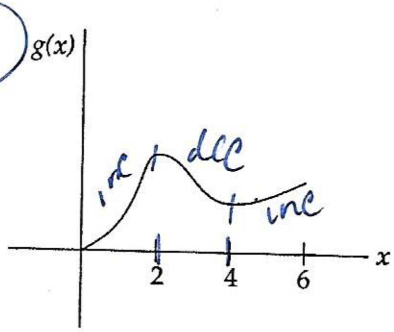
(A)



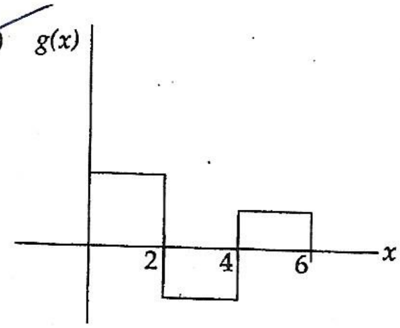
(D)



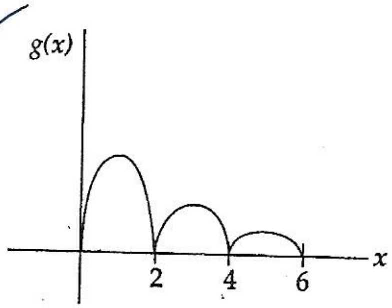
(B)



(E)



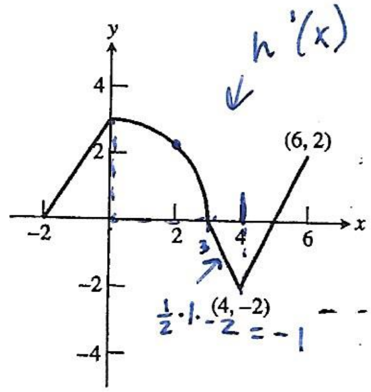
(C)



FREE RESPONSE. No calculator may be used for Problem 11.

11. The graph of g shown in the figure consists of a quarter-circle and three line segments. Let h be the function defined by

$$h(x) = \int_0^x g(t)\,dt$$



(a) Evaluate $h(4)$.

$\frac{1}{4}\pi r^2 + -1 = \frac{1}{4}\pi(3)^2 + -1 = \frac{9}{4}\pi - 1 = \frac{9}{4}\pi - \frac{4}{4} = \frac{9\pi - 4}{4}$

(b) Find all values of x in the interval $[-2, 6]$ at which h has a relative minimum. Justify your answer.

h has a relative minimum at $x=5$ because $h'(x)$ changes from negative to positive at $x=5$.

(c) Find the value of $h'(2)$.

$h'(2) \approx 2.2$

$x^2 + g^2 = 3^2$

$2^2 + g^2 = 9$

$g^2 = 5$

$g = \sqrt{5} \approx 2.24$

(d) Find the x-coordinate of each point of inflection of the graph of h on the interval $(-2, 6)$. Justify your answer.

h has points of inflection at $x=0$ because h' changes from increasing to decreasing there, and also at $x=4$ because h' changes from decreasing to increasing there.