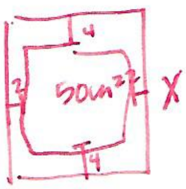
CALC AB
Review 4 Notes: Applications of the derivative
Date:

Optimization:

maximizing/minimizing something

Strategy: (pg 223)

1- Understand the problem
2- Develop a mathematical model of the problem (pictures)
3- Graph function
4- Identify critical points & endpoints ($f'(x)=0$ or $f'(x)$ DNE)
5- Solve the mathematical model
6- Interpret the solution



You are designing a rectangular poster to contain 50 in^2 of printing with a 4 inch margin at the top and bottom and a 2 inch margin on each side. What overall dimensions will minimize the amount of paper used?

minimize: $A = xy$

MUST: $x > 8$

Printed area = 50 means $(x-8)(y-4)=50$

so solve for y:

$\frac{50}{x-8} + 4 = y$

and

$A(x) = x\left(4 + \frac{50}{x-8}\right)$

$A(x) = 4x + \frac{50x}{x-8}$

$A'(x) = 4 + \frac{(x-8)\cdot 50 - 50x\cdot 1}{(x-8)^2} =$

$A'(x) = 4 + \frac{50x - 400 - 50x}{(x-8)^2} = 4 + \frac{400}{(x-8)^2}$

$0 = 4 - \frac{400}{(x-8)^2}$

$400 = 4(x^2 - 16x + 64)$

$100 = x^2 - 16x + 64$

$0 = x^2 - 16x - 36$

$0 = (x-18)(x+2)$

$x = 18$, ~~-2~~ $x > 8$

graph

$\boxed{x = 18 \quad y = 9}$

The height of an object moving vertically is given by $s = -16t^2 + 96t + 112$. Find the object's maximum height and when it occurs.

$s = -16(t^2 - 6t - 7) = -16(t-7)(t+1)$

$t = 7, -1$

$\frac{7 + -1}{2} = 3 = t$

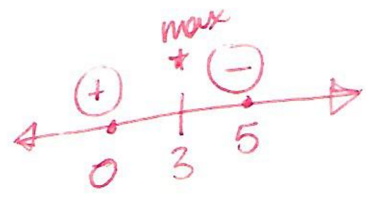
$s(3) = 256$

$s'(t) = -32t + 96$

$s'(t) = -32(t-3)$

$0 = t - 3$

$\boxed{t = 3}$



Related rates:

Strategy: (pgs 250-257)

1- Understand the problem

2- Develop a mathematical model of the problem

3- Write equation relating the variable whose rate of change you seek with the variable(s) whose rate of change you know.

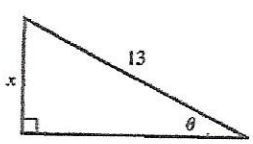
4- Differentiate both sides of the equation implicitly with respect to time t.

5- Substitute values for any quantities that depend on time.

6- Interpret the solution.

No calculator

1. In the right triangle shown, θ is increasing at a constant rate of 2 radians per minute. In units per minute, at what rate is x increasing when $x = 12$?



$\frac{d\theta}{dt} = 2$ rad/min

$\frac{dx}{dt} = ?$

(A) 2 (B) 4 (C) 5 (D) 10 (E) 24

$\sin\theta = \frac{x}{13}$

$\sin\theta = \frac{12}{13}$

$\theta = 67.4°$

$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{13} \cdot \frac{dx}{dt}$

$13(\cos 67.4)(2) = \frac{dx}{dt}$

$9.99 = \frac{dx}{dt}$

$\frac{S}{6} = \frac{6x^2}{6}$ $V = x^3$

Calculator allowed. $x^2 = \frac{S}{6}$ $x = \sqrt{\frac{S}{6}}$

$dx/dt = 0.2$

3. The side of a cube is increasing at a constant rate of 0.2 centimeter per second. In terms of the surface area, S, what is the rate of change of the volume of the cube in cubic centimeters per second? FIND $\frac{dV}{dt}$

(A) 0.1 S (B) 0.2 S (C) 0.6 S (D) 0.04 S (E) 0.008 S

$V = x^3$

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$\frac{dV}{dt} = 3\sqrt{\frac{S}{6}}^2 \cdot 0.2 = \frac{3}{6} \cdot S(0.2)$

$\frac{dV}{dt} = 0.1S$

? → 4. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

dec: $w > l$ $\frac{dl}{dt} = -2$

inc: $l > w$ $\frac{dw}{dt} = 2$

(A) A is always increasing.

(B) A is always decreasing.

(C) A is increasing only when $l > w$.

(D) A is increasing only when $l < w$.

(E) A remains constant.

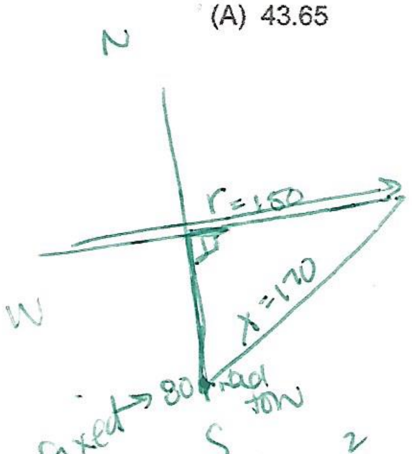
$A = lw$

$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + \frac{dw}{dt} \cdot l$

$\frac{dA}{dt} = -2w + 2l$

5. Two roads cross at right angles, one running north/south and the other east/west. Eighty feet south of the intersection is an old radio tower. A car traveling at 50 feet per second passes through the intersection heading east. At how many feet per second is the car moving away from the radio tower 3 seconds after it passes through the intersection?

(A) 43.65 (B) 44.12 (C) 44.59 (D) 56.67 (E) 81.76



Find $\frac{dx}{dt}$?

$\frac{dr}{dt} = 50$

$80^2 + r^2 = x^2$

$0 + 2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt}$

$2(150) \cdot 50 = 2(170) \cdot \frac{dx}{dt}$

$\frac{15000}{340} = \frac{dx}{dt}$

$44.1176 = \frac{dx}{dt}$

$80^2 + 150^2 = x^2$

$x = \sqrt{28900}$

$x = 170$