NOT in your book

Answer these questions to help you review.
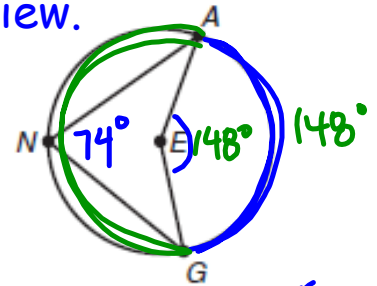
5. In circle E shown, $m\angle ANG = 74°$.

a. Determine $m\angle AEG$.

$$74(2) = 148°$$

b. Determine $m\overset{\frown}{ANG}$.

$$360 - 148 = 212°$$

6. In circle H shown, $m\overset{\frown}{CA} = 105°$, $m\overset{\frown}{EA} = 47°$, and $m\overset{\frown}{ET} = 100°$.

a. Determine $m\angle ETC$.

Intercepted arc is EC $\frac{1}{2}(47 + 105) = 76°$

b. Determine $m\angle TCE$.

Intercepted arc is $\overset{\frown}{TE}$

$$m\overset{\frown}{TE} = 100°$$

$$m\angle TCE = \frac{1}{2} \cdot 100$$

$$\boxed{m\angle TCE = 50°}$$
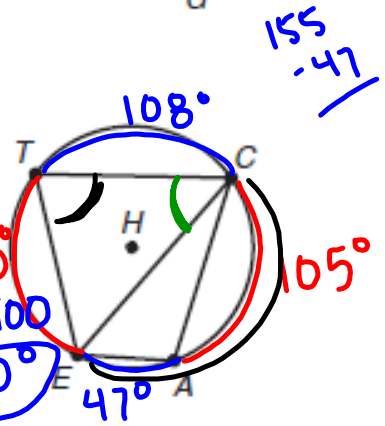
c. Determine $m\angle CAE$.

Intercepted arc is $\overset{\frown}{CE}$

$$m\overset{\frown}{CTE} = 208$$

$$m\angle CAE = \frac{1}{2} \cdot 208$$

$$\boxed{m\angle CAE = 104°}$$

d. Determine $m\angle TEA$.

Intercepted arc is $\overset{\frown}{TA}$

$$m\overset{\frown}{TCA} = 108 + 105$$
$$= 213°$$

$$m\angle TEA = \frac{1}{2} \cdot 213°$$

$$\boxed{m\angle TEA = 106.5°}$$

155 .47

108°

105°

108

100

105
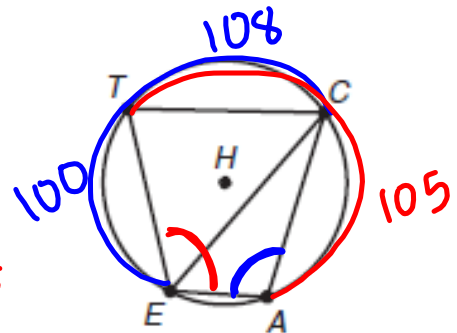
NOT in your book

Use circle S to answer each question. Explain your reasoning.
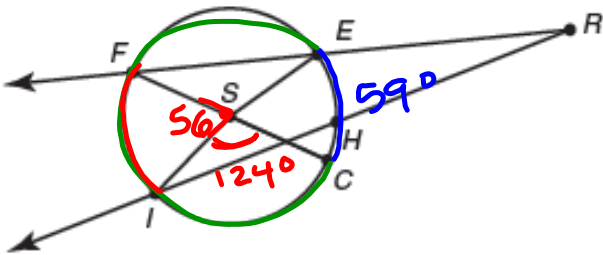
1. Suppose that $m\widehat{CE} = 59°$. What is $m\widehat{CFE}$?

$$360° - 59° = \underline{\underline{301°}}$$

2. Suppose that $m\angle CSI = 124°$. What is $m\widehat{FI}$?
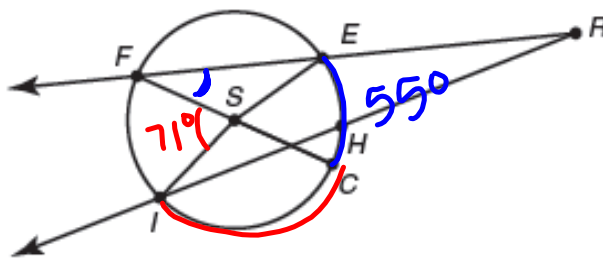
$$180 - 124 = 56° \qquad m\,\widehat{FI} = 56°$$

3. Suppose that $m\widehat{CE} = 55°$. What is $m\angle EFC$?

$$m\,\widehat{CE} = 55 \qquad m\angle EFC = \tfrac{1}{2}(55) = \underline{\underline{27.5°}}$$

4. Suppose that $m\angle FSI = 71°$. What is $m\widehat{IC}$?

$$180 - 71 = 109° \qquad m\,\widehat{IC} = 109°$$

Manhole Covers
Measuring Angles Inside and Outside of Circles

9.3

pg.675 & 677 in your book

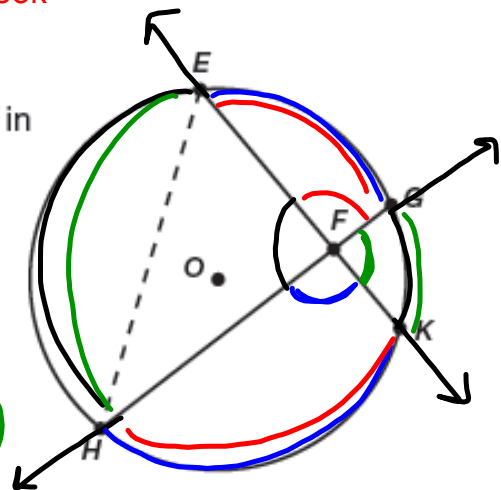2. Prove the Interior Angles of a Circle Theorem.

Given: Chords *EK* and *GH* intersect at point *F* in circle *O*.

Prove: $m\angle KFH = \frac{1}{2}(m\overset{\frown}{HK} + m\overset{\frown}{EG})$

$m\angle EFG = \frac{1}{2}(m\overset{\frown}{HK} + m\angle EG)$

$m\angle EFH = \frac{1}{2}(m\overset{\frown}{EH} + m\overset{\frown}{GK})$

$m\angle GFK = \frac{1}{2}(m\overset{\frown}{GK} + m\overset{\frown}{EH})$

The **Interior Angles of a Circle Theorem** states: "If an angle is formed by two intersecting chords or secants of a circle such that the vertex of the angle is in the interior of the circle, then the measure of the angle is half of the sum of the measures of the arcs intercepted by the angle and its vertical angle."

pg.682 in your book

The **Exterior Angles** of a Circle Theorem states: "If an angle is formed by two intersecting

tangents

~~or secants~~ of a circle such that the vertex of the angle is in the exterior of the circle,

then the measure of the angle is half of the difference of the measures of the arcs

intercepted by the angle."   pg.680 in your book

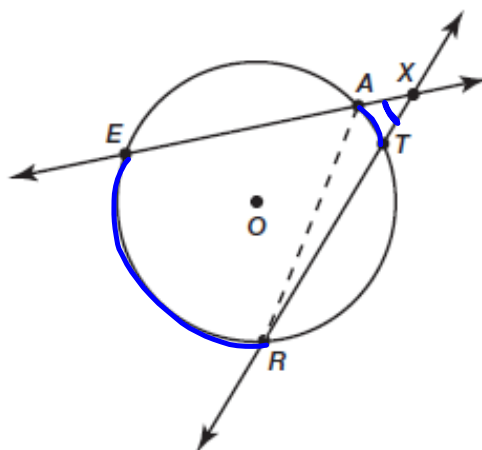3. Prove each case of the Exterior Angles of a Circle Conjecture.

     a. Case 1
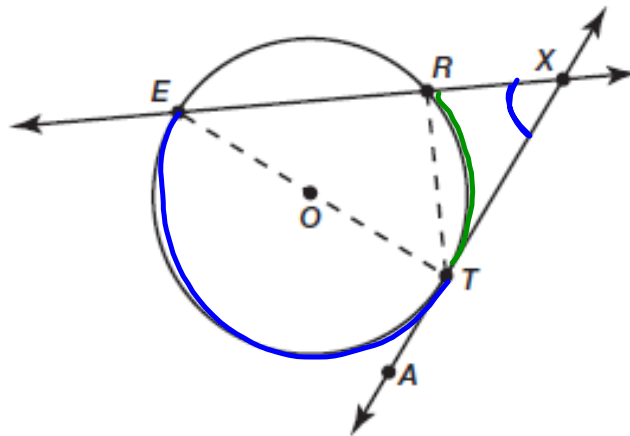
Given: Secant *EX* and tangent *TX* intersect at point *X*.

Prove: $m\angle EXT = \frac{1}{2}(m\overset{\frown}{ET} - m\overset{\frown}{RT})$

Jan 4-9:10 AM

pg.681 in your book

b. Case 2

Given: Secants EX and RX intersect at point X.

Prove: $m\angle EXR = \frac{1}{2}(m\overset{\frown}{ER} - m\overset{\frown}{AT})$

pg.682 in your book

c. Case 3
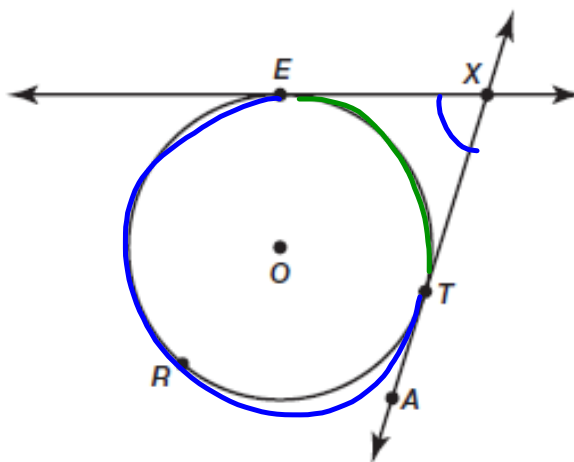
Given: Tangents EX and AX intersect at point X.

Prove: $m\angle EXT = \frac{1}{2}(m\overset{\frown}{ERT} - m\overset{\frown}{ET})$
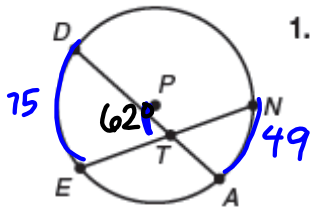
Jan 4-9:16 AM

pg.684 in your book

The **Tangent to a Circle Theorem** states: "A line drawn tangent to a circle is perpendicular to a radius of the circle drawn to the point of tangency."

pg.683 in your book

Jan 4-9:17 AM
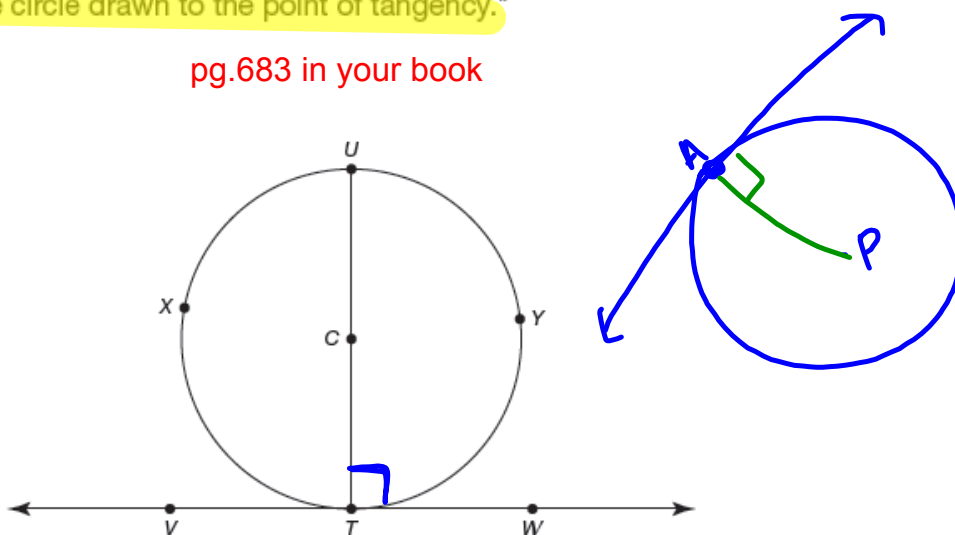
NOT in your book

Interior $\angle$s of a Circle Theorem

1. In circle P shown, $m\widehat{DE} = 75°$ and $m\widehat{NA} = 49°$. Determine $m\angle DTE$.

$$m\angle DTE = \tfrac{1}{2}\left(m\widehat{DE} + m\widehat{NA}\right)$$

$$m\angle DTE = \tfrac{1}{2}(75 + 49)$$

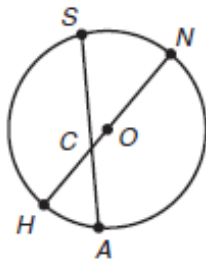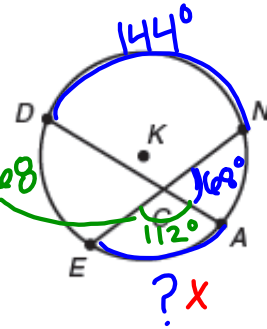$$m\angle DTE = \tfrac{1}{2}(124)$$

$$m\angle DTE = 62°$$

Circle P: points D, N, E, A with center P, intersection at T. Labels: 75, 62, 49.

2. In circle K shown, $m\widehat{DN} = 144°$ and $m\angle NCA = 68°$. Determine $m\widehat{EA}$.

$$m\angle ACE = \tfrac{1}{2}\left(m\widehat{AE} + m\widehat{DN}\right)$$

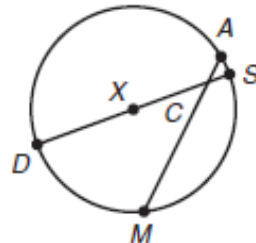$$2 \cdot 112° = 2 \cdot \tfrac{1}{2}(x + 144)$$

$$224 = x + 144$$
$$-144 \qquad -144$$
$$\overline{80° = x} \quad \rightarrow \quad m\widehat{EA} = 80°$$

Circle K: labels 144°, 180-68, 112°, 68°, 112°, ?x, points D, N, K, E, C, A.

3. In circle O shown, $m\widehat{SN} = 55°$ and $m\widehat{HA} = 35°$. Determine $m\angle SCh$.

Circle O: points S, N, C, O, H, A.

4. In circle X shown, $m\widehat{AS} = 11°$ and $m\widehat{MS} = 104°$. Determine $m\angle DCM$.

Circle X: points A, S, X, C, D, M.

Exterior $\angle$s of a circle thm. case 2
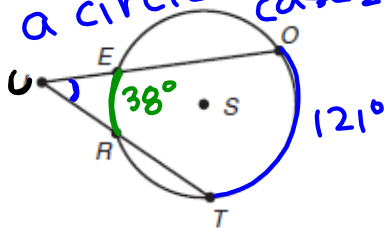
5. In circle S shown, $m\widehat{ER} = 38°$ and $m\widehat{OT} = 121°$. Determine $m\angle OUT$.

$$m\angle OUT = \tfrac{1}{2}\left(m\widehat{OT} - m\widehat{ER}\right)$$

$$m\angle OUT = \tfrac{1}{2}(121 - 38)$$

$$m\angle OUT = \tfrac{1}{2}(83)$$

$$m\angle OUT = 41.5°$$

Circle S: points U, E, O, S, R, T. Labels 38°, 121°.

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Ext. ∠s of a circle
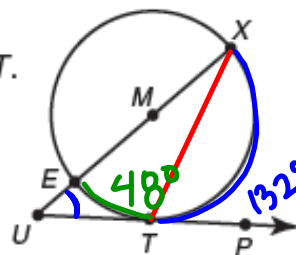case 1

NOT in your book

6. In circle M shown, $\overline{XE}$ is a diameter of the circle and $m\overset{\frown}{XT} = 132°$.
   Draw a chord that connects points X and T. Then determine $m\angle XUT$.

$m\overset{\frown}{ET} = 180 - 132 = 48°$

$m\angle XUT = \frac{1}{2}(m\overset{\frown}{XT} - m\overset{\frown}{ET})$

$m\angle XUT = \frac{1}{2}(132 - 48)$

$m\angle XUT = 42°$

Ext. ∠s of a circle case 3

8. In circle B shown, $m\overset{\frown}{HE} = 99°$.

   $360 - 99 = 261°$

   a. Determine $m\angle HUE$.

   $m\angle HUE = \frac{1}{2}(m\overset{\frown}{HRE} - m\overset{\frown}{HE})$

   $m\angle HUE = \frac{1}{2}(261 - 99)$

   $m\angle HUE = \frac{1}{2}(162)$

   $m\angle HUE = 81°$
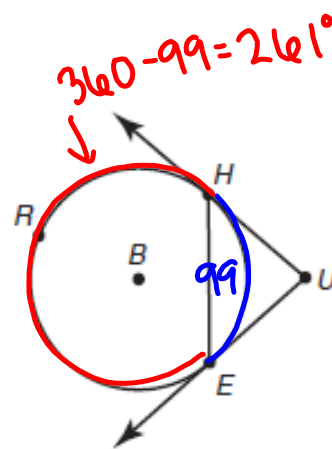
7. In circle G shown, $OH = ES$, $m\overset{\frown}{OH} = 41°$, and
   $m\overset{\frown}{HE} = 171°$. Determine $m\angle EUH$.

   b. Determine $m\angle BHU$.

9. In circle T shown, $m\angle RCE = 57°$ and $m\overset{\frown}{RE} = 141°$.
   Determine $m\overset{\frown}{BL}$.

Jan 4-9:17 AM
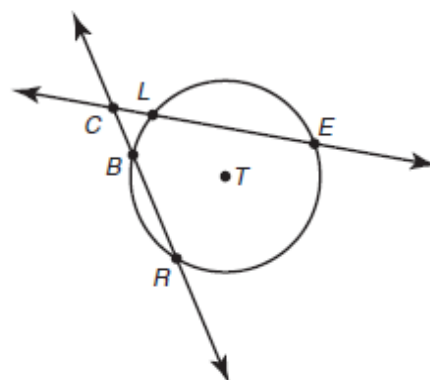
Homework
Finish Lesson 9.3

Aug 24-2:00 PM