Questions on 8.3?

Answer the questions below in your notes for review. This is NOT in your book.

$$S_n = \frac{g_n(r) - g_1}{r-1}$$

Use Euclid's Method to compute each series.

$r = -4$

1 + (−4) + 16 + (−64) + 256 + (−1024)

$$S_n = \frac{-1024(-4) - 1}{-4-1} = \frac{4096-1}{-5} = \boxed{-819}$$

$\cdot 3 \quad \cdot 3$

2 + 6 + 18 + 54 + 162

$$S_n = \frac{162(3) - 2}{3-1} = \boxed{242}$$

Dec 10-3:22 PM

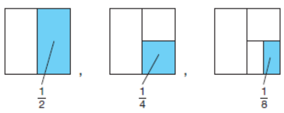
8.4 These Series Just Go On . . . And On . . . And On . . .

Infinite Geometric Series

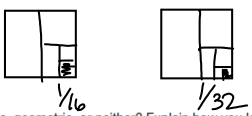
pg.607-608 in your book.

Previously, you calculated sums of finite series. What if a series was infinite? Let's see if there is a way to calculate the sum of an infinite series.

The first three terms of an infinite sequence are represented by the figures shown. In this sequence, each square represents a unit square, and the shaded part represents area.



1. Sketch the next two figures to model this sequence, and write the numbers that correspond to each term.



2. Is this sequence arithmetic, geometric, or neither? Explain how you know. If possible, write an explicit formula for the sequence.

geometric

$2^3 \cdot 2^4 = 2^{3+4} = 2^7$

$g_n = g_1 \cdot r^{n-1}$

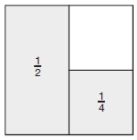
$g_n = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} = \frac{1}{2}^{\cancel{1}+n\cancel{-1}} = \frac{1}{2}^n$

$$\boxed{g_n = \frac{1}{2}^n}$$

Nov 19-8:31 AM

pg.609 in your book

3. Consider the series, or sum, of the first two terms of this infinite sequence. The sum of the first two terms can be modeled with a diagram, as shown.



$\frac{3}{4}$

Continue shading the diagram to represent the sum of the first five terms of the series. What happens to the total area that is shaded every time you shade another piece of the unit square?

5 mins to finish

4. In the table shown, n represents the term number of the series, and S_n represents the sum of the first n terms of the series. Use the sequence from the unit square in Question 3 to answer each question.

n	1	2	3	4	5	10	25
S_n as a Fraction	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{1023}{1024}$	$\frac{33,554,431}{33,554,432}$
S_n as a Decimal	0.5	0.75	0.875	0.93875	0.96875	0.99902...	0.999...

a. Complete the table for $n = 1$ through $n = 5$ to show the sum of the series that corresponds to the previous diagram. Write each sum as a fraction and as a decimal.

b. Describe the pattern you see in the table.

*numerator is one less than the denominator

*denominator is 2^n

c. Use the pattern to complete the table for the final two columns.

5. What value does the series approach as n gets greater?

The series approaches 1.

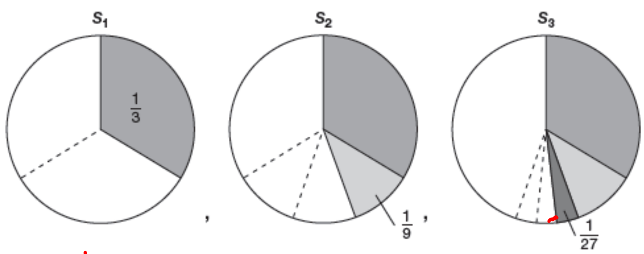
Nov 19-8:32 AM

pg.611 in your book

In the previous problem, you saw how an infinite geometric series can have a finite sum. Let's see if this is the case for *any* infinite geometric series.

1. Examine the given formula and accompanying diagram for each infinite geometric series. Identify both r and g_1 for each series. Then, determine if the sum is infinite or finite. If the sum is finite, estimate it.

a. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots = \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^i$



$r =$ $\frac{1}{3}$

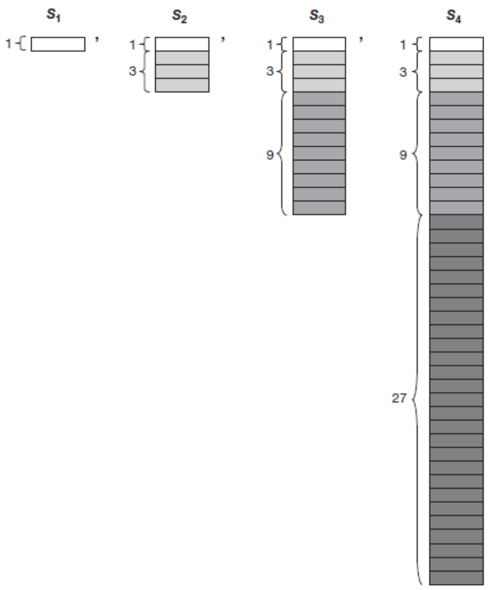
$g_1 =$ $\frac{1}{3}$

$S =$ $1/2$

Oct 9-8:12 AM

pg.610 in your book

b. $1 + 3 + 9 + 27 + \cdots = \sum_{i=1}^{\infty} \frac{1}{3}(3)^i$



$r =$ 3

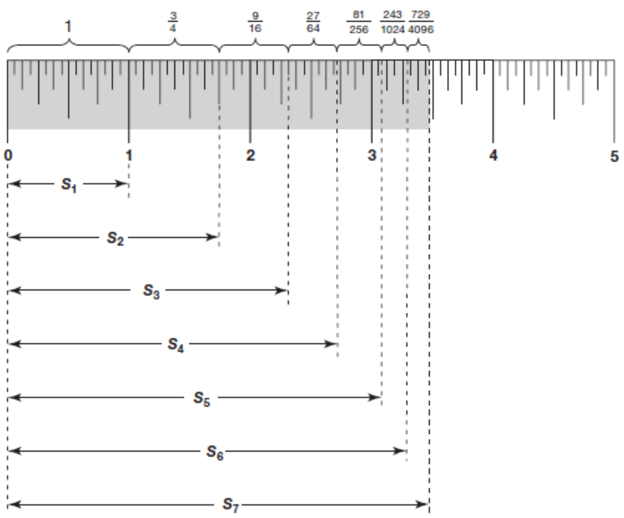
$g_1 =$ 1

$S =$ ∞ (infinite sum)

Jan 12-12:54 PM

pg.611 in your book
take 5 mins to finish c - e on pgs.613-614

c. $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \frac{81}{256} + \frac{243}{1024} + \frac{729}{4096} + \cdots = \sum_{i=1}^{\infty} \frac{4}{3}\left(\frac{3}{4}\right)^i$



$r =$ 3/4

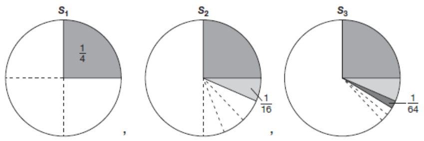
$g_1 =$ 1

$S =$ 4

Jan 8-9:01 AM

pg.611 in your book

d. $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^i$

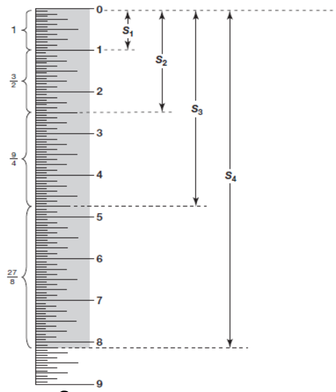


$r =$ 1/4

$g_1 =$ 1/4

$S =$ 1/3

e. $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \cdots = \sum_{i=1}^{\infty}\frac{2}{3}\left(\frac{3}{2}\right)^i$



$r =$ 3/2

$g_1 =$ 1

$S =$ ∞ (infinite sum)

Jan 14-9:04 AM

pg.614 in your book

2. Analyze the common ratio for each series in Question 1.

a. What do you notice about the series with infinite sums?

Common ratio is greater than 1.

b. What do you notice about the series with finite sums?

Common ratio is between 0 & 1.

pg.615 in your book

A **convergent series** is an infinite series that has a finite sum. A **divergent series** is an infinite series that does not have a finite sum. If a series is divergent, the sum is infinity.

infinite sum.

The formula to compute a convergent geometric series S is shown.

$$\boxed{S = \frac{g_1}{1 - r}}$$

Notice that S denotes the sum of an *infinite* series. This notation should not be confused with S_n, which represents the sum of the nth term of a series.

3. Consider each infinite geometric series from Question 1. Determine whether each series is convergent or divergent, and explain how you know. If a series is convergent, use the formula to compute the sum. If a series is divergent, write infinity.

a. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots = \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^i$

Convergent or divergent? convergent

Explanation:

$$S = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \frac{1}{2}$$

$S =$ 1/2

Jan 14-9:05 AM

pg.615 in your book

b. $1 + 3 + 9 + 27 + \cdots = \sum_{i=1}^{\infty} \frac{1}{3}(3)^i$

Convergent or divergent? divergent

Explanation: Has a common ratio of 3, which is greater than 1.

$S =$ ∞ (infinity)

pg.616 in your book

c. $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \frac{81}{256} + \frac{243}{1024} + \frac{729}{4096} + \cdots = \sum_{i=1}^{\infty} \frac{4}{3}\left(\frac{3}{4}\right)^i$

Convergent or divergent? convergent

Explanation:

$$S = \frac{1}{1-\frac{3}{4}} = \frac{1}{\frac{1}{4}} = 4$$

$S =$ 4

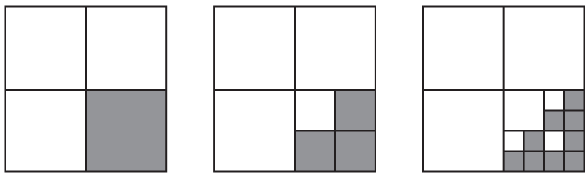
**take 5 mins to finish pgs. 616-617 in your book

d) $S = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{3} = \frac{1}{3}$

Jan 14-9:07 AM

NOT in your book

1. The first three terms of an infinite geometric sequence are represented by the figures shown.



$\frac{1}{4}$ $\frac{3}{16}$ $\frac{9}{64}$

a. Determine g_1 and r for the sequence. $r = \frac{3}{4}$; $g_1 = \frac{1}{4}$

b. Determine whether the series is convergent or divergent. Explain your reasoning.

c. Compute the series. $S = \frac{g_1}{1-r} = \frac{\frac{1}{4}}{1-\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{1}{4} \div \frac{1}{4} = 1$

2. Consider the geometric series $\frac{1}{100} + \frac{2}{100} + \frac{4}{100} + \frac{8}{100} + \cdots$ $r = 2$

a. Determine whether the series is convergent or divergent. Explain your reasoning.

divergent ; $r > 1$

b. Compute the series.

∞

Jan 14-9:07 AM

NOT in your book

3. The first three terms of an infinite geometric sequence are represented by the figures shown.

$\frac{3}{4}$ $\frac{9}{16}$ $\frac{27}{64}$

a. Determine g_1 and r for the sequence. $r = \frac{3}{4}$; $g_1 = \frac{3}{4}$

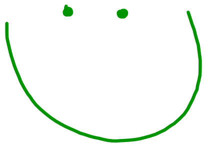
b. Determine whether the series is convergent or divergent. Explain your reasoning. convergent $0 < r < 1$

c. Compute the series. $S = \frac{\frac{3}{4}}{1 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$

4. Consider the geometric series $\frac{1}{5} + \frac{10}{45} + \frac{100}{405} + \ldots$ $r = \frac{10}{9}$; $g_1 = \frac{1}{5}$

a. Determine whether the series is convergent or divergent. Explain your reasoning. divergent; $r > 1$

b. Compute the series.



Jan 14-9:07 AM

Homework
Finish Lesson 8.4

Oct 21-8:47 AM