Questions on 8.3c HW?

6

$d = \sqrt{2} \cdot s$

$\frac{d}{\sqrt{2}} = \frac{\sqrt{2} \cdot s}{\sqrt{2}}$

$\frac{d}{\sqrt{2}} = s$

$s = \frac{2\sqrt{1-x^2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2(1-x^2)}}{\cancel{2}}$

$s = \sqrt{2(1-x^2)}$

$A(x) = \sqrt{2(1-x^2)}^{2}$

$A(x) = 2(1-x^2)$

$V = \int_{-1}^{1} [2(1-x^2)]\,dx =$

$2\left[x - \frac{x^3}{3}\right]_{-1}^{1}$

$2\left[\left(1 - \frac{1}{3}\right) - \left(-1 - \frac{-1}{3}\right)\right] =$

$2\left[\frac{2}{3} + \frac{2}{3}\right] = \frac{8}{3}$ units^3

8.4 Integrals as Net Change (8.1 in book)

Example - Linear Motion Revisited

$v(t) = 10 - 2t$ is the velocity in m/sec of a particle moving along the x-axis when $0 \le t \le 9$. Use analytic methods to:

$10 - 2t = 0$
$t = 5$

(a) Determine when the particle is moving to the right, to the left, and stopped.

$+$ 0 $-$
0 5 9

Right: $0 \le t < 5$
Left: $5 < t \le 9$
Stopped: $t = 5$

(b) Find the particle's displacement for the given time interval.

(c) If $s(0) = 3$, what is the particle's final position?

initial position + displacement
$3 + 9 = 12$ m

(d) Find the total distance traveled by the particle.

(b) $\int_0^9 (10 - 2t)\,dt = \left[10t - \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^9 = [(90 - 81) - 0] = 9$ m to the right

(a) When $v(t) > 0$, the particle is moving right. This occurs when $0 \le t < 5$. When $v(t) = 0$, the particle is stopped. This occurs when $t = 5$. When $v(t) < 0$, the particle is moving left. This occurs when $5 < t \le 9$.

(R) pos. $10 - 2t$

(L) neg. $-10 + 2t$

(d) Total Dist: $\int_0^9 |10 - 2t|\,dt$

(+)/(R) (-)/(L)

$\int_0^5 (10 - 2t)\,dt + \int_5^9 (2t - 10)\,dt$

$[10t - t^2]_0^5 + [t^2 - 10t]_5^9 =$

$[50 - 25] + [(81 - 90) - (25 - 50)]$

$25 + -9 + 25 = \underline{\underline{41\text{ m}}}$

(b) displacement $= \int_0^9 v(t)\,dt$

$= \int_0^9 (10 - 2t)\,dt$

$= \left[10t - t^2\right]_0^9$

$= 90 - 81$

$= 9$

During the first 9 seconds of motion, the particle moves 9 m to the right.

(c) It starts at $s(0) = 3$, so its position at $t = 9$ is

New position = initial position + displacement

$= 3 + 9.$

(d) Total distance $= \int_0^9 |v(t)|\,dt = \int_0^9 |10 - 2t|\,dt$

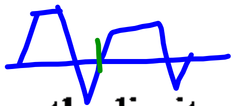
$$= \int_0^5 (10 - 2t)\,dt + \int_5^9 (2t - 10)\,dt$$

$$= \left[10t - t^2\right]_0^5 + \left[t^2 - 10t\right]_5^9$$

$$= 50 - 25 + 81 - 90 - 25 + 50 = 41\text{ m}$$

Strategy for Modeling with Integrals

1. Approximate what you want to find as a Riemann sum of values of a continuous function multiplied by interval lengths. If $f(x)$ is the function and $[a,b]$ the interval, and you partition the interval into subintervals of length Δx, the approximating sums will have the form $\sum f(c_k)\Delta x$ with c_k a point in the kth subinterval.



2. Write a definite integral, here $\int_a^b f(x)dx$, to express the limit of these sums as the norms of the partitions go to zero.
3. Evaluate the integral numerically or with an antiderivative.

Example

From 1970 to 1980, the rate of potato consumption in a particular country was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1975?

end of 70 - 1 73-4
71 - 2* 74-5
72 - 3 75-6

$\int_2^6 (2.2 + 1.1^t)dt$

≈ 14.692 million bushels

We seek the cummulative effect of the consumption rate for $2 \le t \le 6$.
Step 1: Reimann sum: We partition $[2,6]$ into subintervals of length Δt and let t_k be a time in the kth subinterval. The amount consumed during this interval is approximately $C(t_k)\Delta t$ million bushels. The consumption for $2 \le t \le 6$ is approximately $\sum C(t_k)\Delta t$ million bushels.

Step 2: Definite integral: The amount consumed from $t = 2$ to $t = 6$ is the limit of these sums as the norms of the partitions go to zero $\int_2^6 C(t)dt = \int_2^6 \left(2.2 + 1.1^t\right)dt$ million bushels.

Step 3: Evaluate: Evaluating numerically, we obtain

$\text{NINT}(2.2 + 1.1^t, t, 2, 6) \approx 14.692$ million bushels.

Feb 28-4:25 PM

Homework

8.4 WKS