Questions on 8.3b HW? Integral quiz soon...

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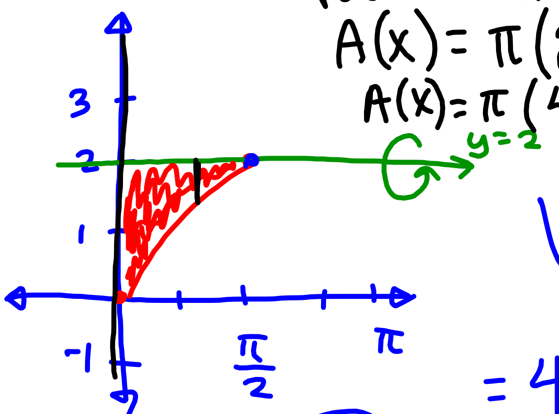
cross-sections: circles w/ radius: $2-2\sin x$

$A(x) = \pi(2-2\sin x)^2$

$A(x) = \pi(4 - 8\sin x + 4\sin^2 x)$



$$V = 4\pi\int_0^{\pi/2}(1 - 2\sin x + \sin^2 x)\,dx =$$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$$= 4\pi\int_0^{\pi/2}\left(1 - 2\sin x + \frac{1-\cos(2x)}{2}\right)dx =$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

$$= 4\pi\left[\left(x + 2\cos x + \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\cdot\sin 2x\right]_0^{\pi/2}$$

$$4\pi\left[\frac{3}{2}x + 2\cos x - \frac{\sin(2x)}{4}\right]_0^{\pi/2} =$$

$$4\pi\left[\left(\frac{3}{2}\cdot\frac{\pi}{2} + 2\overset{0}{\cancel{\cos\left(\frac{\pi}{2}\right)}} - \frac{\overset{0}{\cancel{\sin\pi}}}{4}\right) - \left(0 + 2\overset{1}{\cancel{\cos 0}} - \frac{\overset{0}{\cancel{\sin 0}}}{4}\right)\right]$$

$$4\pi\left[\frac{3\pi}{4} - 2\right] = 3\pi^2 - 8\pi \approx 4.48$$

units^3

Nov 3-2:38 PM

8.3 Volumes of Solids of Known Cross Sections

Volume of a Solid

The definition of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$V = \int_a^b A(x)dx.$

How to find volumes by the method of slicing...

1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

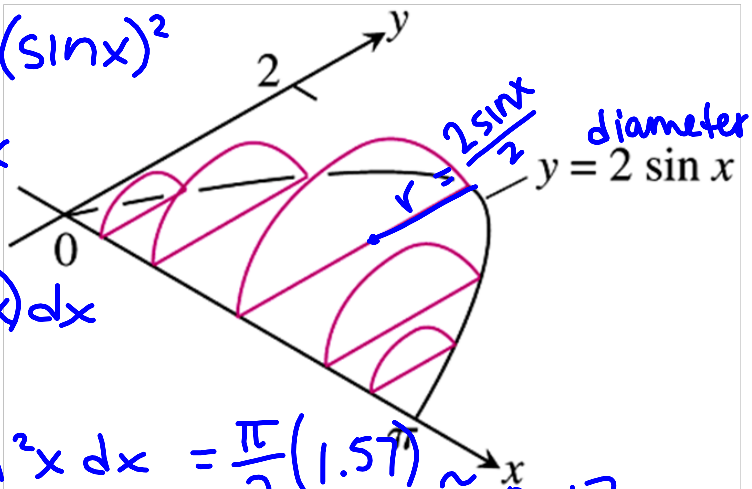
Example - Other Cross Sections

A solid is made so that its base is the shape of the region between the x-axis and one arch of the curve $y = 2\sin x$. Each cross section cut perpendicular to the x-axis is a semicircle whose diameter runs from the x-axis to the curve. Find the volume of the solid.

$r = \frac{2\sin x}{2} = \sin x$

$A(x) = \pi \cdot \frac{1}{2} \cdot (\sin x)^2$

$A(x) = \frac{\pi}{2} \sin^2 x$



$V = \int_0^{\pi} \left(\frac{\pi}{2} \cdot \sin^2 x\right) dx$

$V = \frac{\pi}{2} \int_0^{\pi} \sin^2 x \, dx = \frac{\pi}{2}(1.57) \approx 2.47 \text{ units}^3$

The semicircle at each point x has radius $= \dfrac{2\sin x}{2} = \sin x$

and area $A(x) = \dfrac{1}{2}\pi(\sin x)^2$.

$$\text{So, } V = \frac{\pi}{2}\int_0^{\pi} (\sin x)^2 \, dx$$

$$= \frac{\pi}{2} \text{NINT}\left((\sin x)^2, x, 0, \pi\right)$$

$$= \frac{\pi}{2}(1.570796327)$$

$$= \frac{\pi^2}{4} \text{ in}^3.$$

Examples

$x^2 = 2-y$

$x = \sqrt{2-y}$

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

rewrite in terms of y (solve for x)

~~(A)~~ $\pi \int_0^2 (2-y)^2\, dy$

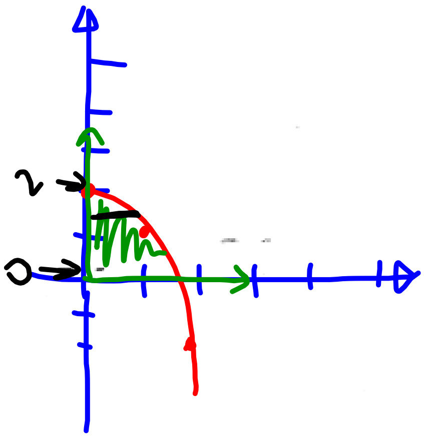
(B) $\int_0^2 (2-y)\, dy$

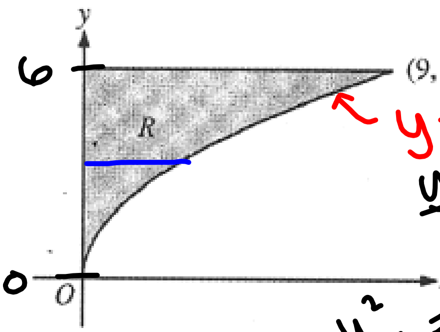
~~(C)~~ $\pi \int_0^{\sqrt{2}} (2-x^2)^2\, dx$

~~(D)~~ $\int_0^{\sqrt{2}} (2-x^2)^2\, dx$

~~(E)~~ $\int_0^{\sqrt{2}} (2-x^2)\, dx$

$V = \int_0^2 \cancel{\sqrt{2-y}}^{\,\cancel{2}}\, dy$





$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

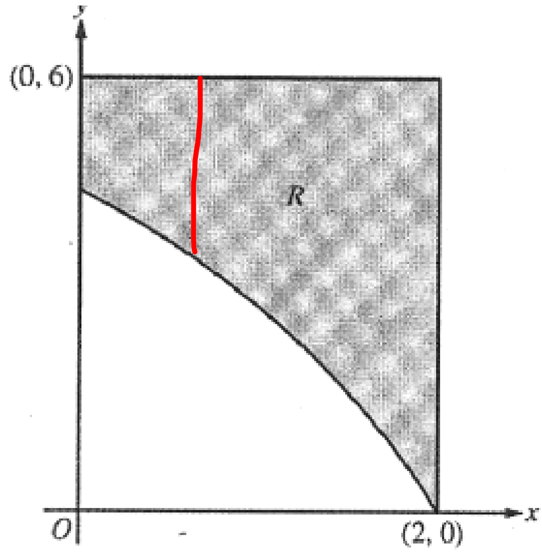
$\frac{y^2}{4} = \left(\frac{y}{2}\right)^2 = x$

Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

$A(x) = \frac{3y^2}{4} \cdot \frac{y^2}{4} = \frac{3y^4}{16}$ height $= 3\left(\frac{y^2}{4}\right)$ Length $= \frac{y^2}{4}$

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$V = \int_0^6 \left(\frac{3y^2}{4}\right)\left(\frac{y^2}{4}\right) dy = \boxed{\int_0^6 \frac{3y^4}{16}\, dy}$$

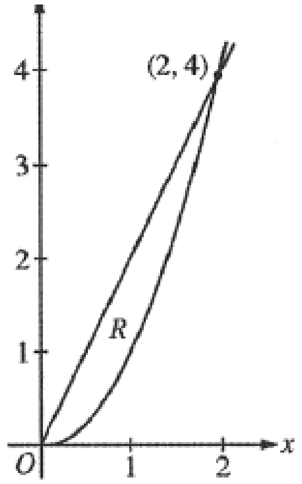


square side length: $6 - 4\ln(3-x)$

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$V = \int_0^2 [6 - 4\ln(3-x)]^2\,dx = 26.27$ units^3



Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

Homework

8.3b: pg.410-412 #1-6,39,42