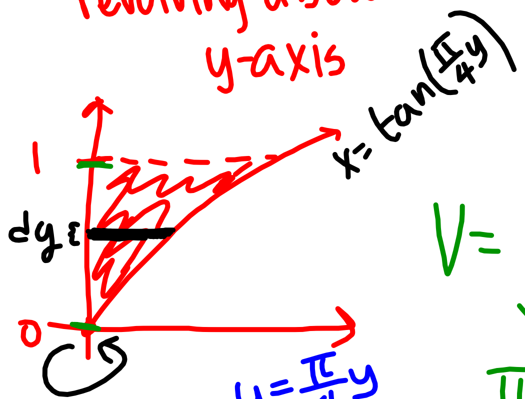
Questions on 8.2 WKS? 8.3 HW?

9 $x = \tan\left(\frac{\pi}{4}y\right)$

revolving about y-axis

cross-sections: circles w/ radius $x = \tan\left(\frac{\pi}{4}y\right)$

Area: $A(x) = \pi\left(\tan^2\left(\frac{\pi}{4}y\right)\right)$



$$V = \int_0^1 \pi \cdot \tan^2\left(\frac{\pi}{4}y\right) dy =$$

$$\pi \int_0^1 \left(\sec^2\left(\frac{\pi}{4}y\right) - 1\right) dy =$$

$u = \frac{\pi}{4}y$

$du = \frac{\pi}{4}dy$

$\frac{4}{\pi} \cdot du = dy$

$$\pi\left[\frac{4}{\pi}\tan\left(\frac{\pi}{4}y\right) - y\right]_0^1$$

$$\pi\left[\left(\frac{4}{\pi} \cdot \tan\frac{\pi}{4} - 1\right) - \left(\cancel{\frac{4}{\pi}\tan 0 - 0}\right)\right] =$$

$$\pi\left[\left(\frac{4}{\pi} \cdot 1 - 1\right)\right] = \boxed{4 - \pi \text{ units}^3}$$

8.3 Volumes of Solids of Revolution about a Line Parallel to an Axis

Volume of a Solid

The definition of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$V = \int_a^b A(x)dx.$

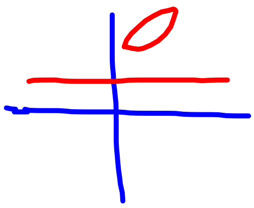
$V = \int_c^d A(y)\,dy$

How to find volumes by the method of slicing...

1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

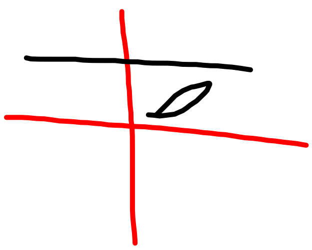
Nov 14-10:50 AM

Axis of revolution is parallel to the x-axis and below the area:

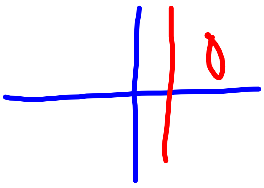


Axis of revolution is parallel to the x-axis and above the area:

$$V = \int_a^b A(x)\,dx$$

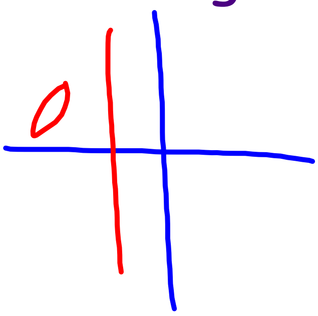


Axis of revolution is parallel to the y-axis and to the left of the area:



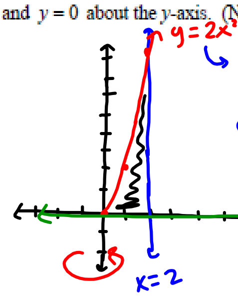
Axis of revolution is parallel to the y-axis and to the right of the area:

$$V=\int_c^d A(y)\,dy$$



Examples

Find the volume of the solid formed by revolving the region bounded by the curves $y = 2x^2$, $x = 2$, and $y = 0$ about the y-axis. (No calculator)



$\frac{y}{2} = \frac{2x^2}{2}$

$\sqrt{\frac{y}{2}} = \sqrt{x^2}$

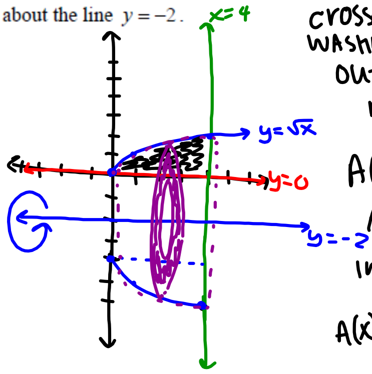
$\sqrt{\frac{y}{2}} = x$

cross-sections:
washers:
outer ⊙:
radius = 2
$A(x) = \pi 2^2 = 4\pi$
inner ⊙:
radius = $\sqrt{\frac{y}{2}}$
$A(x) = \pi\left(\sqrt{\frac{y}{2}}\right)^2 = \frac{\pi y}{2}$

$$V = \int_0^2 \pi\left(4 - \frac{y}{2}\right)dy = \pi\left[4y - \frac{y^2}{4}\right]_0^2 =$$

$$\pi\left[(8-1) - (0)\right] = 7\pi \text{ units}^3$$

Find the volume of the solid generated by revolving the area bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the line $y = -2$.



cross-sections:
WASHERS→
outer radius
$r = \sqrt{x} - -2$
$= \sqrt{x} + 2$
$A(x) = \pi(\sqrt{x}+2)^2$
$A(x) = \pi(x + 4\sqrt{x} + 4)$
inner radius
$r = 0 - -2 = 2$
$A(x) = \pi(2)^2 = 4\pi$

$$V = \int_0^4 \pi\left[(x + 4\sqrt{x} + \cancel{4}) - \cancel{4}\right]dx$$

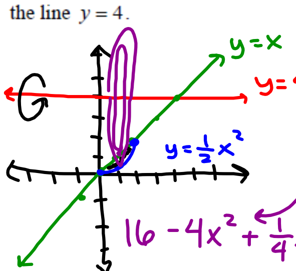
$$= \pi\int_0^4 (x + 4\sqrt{x})\,dx = \pi\left[\frac{x^2}{2} + \frac{24x^{3/2}}{3}\right]$$

$$= \pi\left[\frac{x^2}{2} + \frac{8x^{3/2}}{2}\right]_0^4 =$$

$$= \pi\left[\left(8 + \frac{8}{3}\cdot 4^{3/2}\right) - (0)\right] = \frac{88\pi}{3}$$

≈ 92.15 units³

Find the volume of the solid generated by revolving the area bounded by $y = \frac{1}{2}x^2$ and $y = x$ about the line $y = 4$.



cross-sections:
washers:
outer radius $= 4 - \frac{1}{2}x^2$
$A(x) = \pi\left(4 - \frac{1}{2}x^2\right)^2$
inner radius $= 4 - x$
$A(x) = \pi(4-x)^2$

$16 - 4x^2 + \frac{1}{4}x^4$

$16 - 8x + x^2$

$$V = \int_0^2 \pi\left[\left(\cancel{16} - 4x^2 + \frac{1}{4}x^4\right) - \left(\cancel{16} - 8x + x^2\right)\right]dx =$$

$$V = \pi\int_0^2\left(8x - 5x^2 + \frac{1}{4}x^4\right)dx = \pi\left[\frac{4\cancel{8}x^2}{\cancel{2}} - \frac{5x^3}{3} + \frac{x^5}{20}\right]_0^2$$

$$\pi\left[\left(16 - \frac{40}{3} + \frac{32}{20}\right) - 0\right] = \boxed{4.26\pi \approx 13.041 \text{ units}^3}$$

HW: p411 #11-22

Feb 28-4:25 PM

Homework

8.3b: pg.411 #11-20

Oct 21-8:47 AM