Questions on 8.2 HW?

QUIZ FRIDAY, MARCH 10 ON 15 INTEGRALS YOU MUST MEMORIZE :)

→ Questions on 8.2 WKS - WEDNESDAY!

8.3 Volumes of Solids of Revolution about an Axis

Volume of a Solid

The definition of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

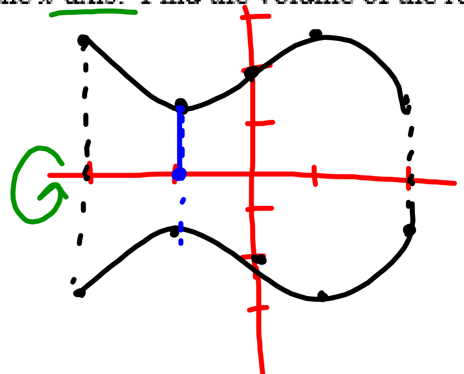
$$V = \int_a^b A(x)dx.$$

How to find volumes by the method of slicing...

1. Sketch the solid and a typical 2-dimensional cross section.
2. Find a formula for $A(x)$. of cross-section
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

Examples

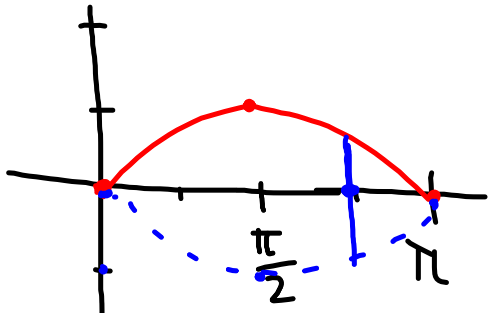
The region between the curve $f(x) = 2 + x\cos x$ and the x-axis, between -2 and 2, is revolved about the x-axis. Find the volume of the resulting solid. (Calculator)



$$V = \int_{-2}^{2} \pi (2 + x\cos x)^2\, dx = 52.43 \text{ units}^3$$

cross-section:
circle w/ radius
of $f(x) = 2 + x\cos x$
and $A(x) = \pi(2 + x\cos x)^2$

Find the volume of the solid formed by revolving the curve $f(x) = \sqrt{\sin x}$ on the interval $(0, \pi)$ about the x-axis. (No calculator)



$$V = \int_0^{\pi} \pi \cdot \sin x\, dx = \pi \int_0^{\pi} \sin x\, dx$$

cross-sections
are circles w/
radius $f(x) = \sqrt{\sin x}$
and area $A(x) = \pi \left(\sqrt{\sin x}\right)^2$
$= \pi \cdot \sin x$

$$\pi \left[-\cos x\right]_0^{\pi} =$$

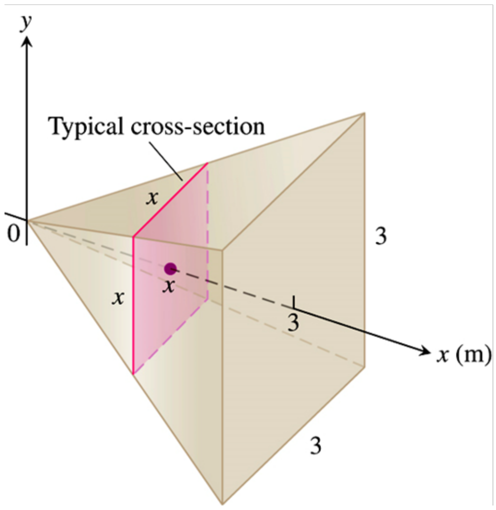
$$\pi(-\cos\pi + \cos 0) =$$

$$\pi(+1 + 1) =$$

$$\boxed{2\pi \text{ units}^3}$$

Example

A pyramid 3 m high has congruent triangular sides and a square base that is 3 m on each side. Each cross section of the pyramid parallel to the base is a square. Find the volume of the pyramid.



1. Sketch: Draw the pyramid with its vertex at the origin and its altitude along the interval $\overset{a}{0} \le x \le \overset{b}{3}$. Sketch a typical cross section at a point x between 0 and 3.

2. Find a formula for $A(x)$: The cross section at x is a square x meters on a side, so $A(x) = \underline{x^2}.$

3. Find the limits of integration: The square goes from $x = 0$ to $x = 3$.

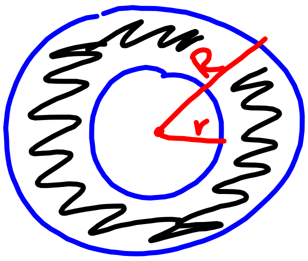
4. Integrate to find the volume:

$$V = \int_0^3 A(x)dx$$
$$= \int_0^3 x^2 dx$$
$$= \frac{x^3}{3}\Big]_0^3$$
$$= 9 \text{ m}^3$$

$$\int_0^3 A(x)\,dx = \int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 =$$
$$\frac{27}{3} - \frac{0}{3} = 9\,m^3$$

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}\cdot(3\times3)\times3$$
$$V = \frac{1}{3}\cdot 27$$
$$V = 9\,m^3$$

Washers



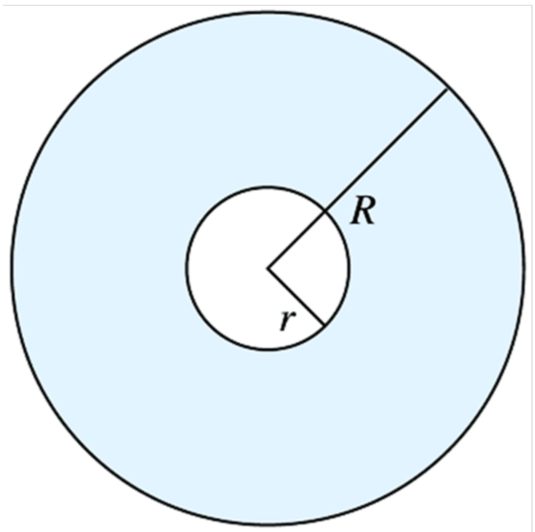
area of washers:
area of outer ⊙ − area of inner ⊙

R = radius of outer circle

r = radius of inner circle

Example

The region between the graph of $f(x) = 2 + x\cos x$ and the x-axis over the interval $[-2,2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.



Revolving the region about the x-axis generates a vase-shaped solid. The cross section at a typical point x is circular, with radius equal to $f(x)$. Its area is $A(x) = \pi\left(f(x)\right)^2$.

The volume of the solid is

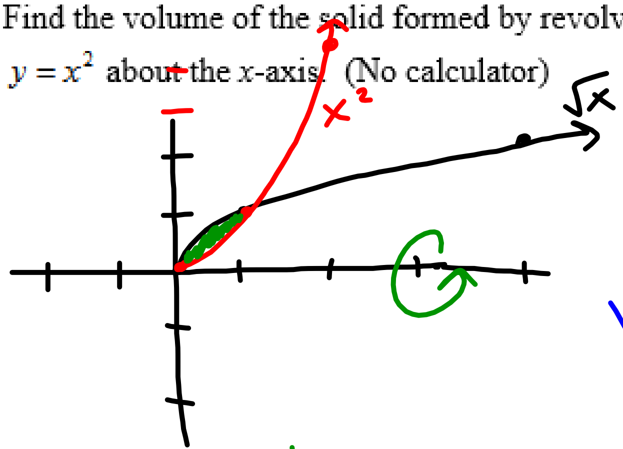
$$V = \int_{-2}^{2} A(x)\,dx$$

$$= \text{NINT}(\pi\left(2 + x\cos x\right)^2, x, -2, 2)$$

$$\approx 52.43 \text{ units cubed.}$$

Examples

Find the volume of the solid formed by revolving the area bounded by the functions $y=\sqrt{x}$ and $y=x^2$ about the x-axis. (No calculator)



Cross sections:
circles -
outer ⊙:
$R=\sqrt{x}$ and
$A(x)=\pi(\sqrt{x})^2$
Inner ⊙:
$r=x^2$
$A(x)=\pi(x^2)^2$

$$V=\int_0^1 \pi\left(\sqrt{x}^2-(x^2)^2\right)dx=$$

$$\pi\int_0^1 (x-x^4)\,dx=$$

$$\pi\left[\frac{x^2}{2}-\frac{x^5}{5}\right]_0^1=$$

$$\pi\left[\left(\frac{1}{2}-\frac{1}{5}\right)-(0-0)\right]=$$

$$\frac{3\pi}{10}\approx 0.94\text{ units}^3$$

Find the volume of the solid formed by revolving the region bounded by the curves $y=2x^2$, $x=2$, and $y=0$ about the y-axis. (No calculator)

Feb 28-4:25 PM

Homework

8.3a: pg.411 #7-10