Questions on 8.2?

Answer the question below in your notes for review. This is NOT in your book.

Use sigma notation to rewrite each finite series. Then, calculate the given sum.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32};\ S_3$$

$$S_3 = \sum_{i=1}^{3} a_i = a_1+a_2+a_3 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \boxed{\frac{7}{8}}$$

$$S_n = \frac{n(a_1+a_n)}{2}$$

$$S_3 = \frac{3\left(\frac{1}{2}+\frac{1}{8}\right)}{2} = \frac{3\left(\frac{5}{8}\right)}{2} = \frac{\frac{15}{8}}{2} = \frac{15}{16}$$

p589 4d

$$\left.\begin{aligned} n &= 100 \\ a_1 &= 2 \\ a_{100} &= 200 \end{aligned}\right\}$$

$$S_{100} = \frac{100(2+200)}{2} = \frac{100(202)}{2} = 10{,}100$$

$$S_{100} = 2+4+6+\cdots+196+198+200$$

$$+\ S_{100} = 200+198+196+\cdots+6+4+2$$

$$2\cdot S_{100} = 202+202+202+\cdots+202+202+202$$

$$\frac{2\cdot S_{100}}{2} = \frac{100(202)}{2}$$

$$S_{100} = \frac{100(202)}{2}$$

I Am Having a Series Craving (For Some Math)!

Geometric Series

8.3

pg.595-596 in your book.

A **geometric series** is the sum of the terms of a geometric sequence. Recall, that the sequence 1, 3, 9, 27, 81 is a geometric sequence because the ratio of any two consecutive terms is constant. Adding the terms creates the geometric series $1 + 3 + 9 + 27 + 81$.

The constant ratio of this geometric sequence is 3 because $\frac{3}{1} = \frac{9}{3} = \frac{27}{9} = \frac{81}{27} = 3$ Recall all geometric sequences have a constant ratio between successive terms.

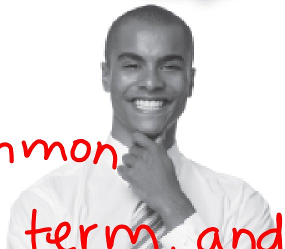
Theresa raises her hand and claims that she has a "trick" for quickly calculating the sum of any geometric series. She asks members of the class to write any geometric series on the board. She boasts that she can quickly tell them how to determine the sum without adding all of the terms. Several examples are shown.

Paul: $r=3$ "OK, so prove it! What is the sum of $1 + 3 + 9 + 27 + 81 + 243 + 729$?" 1093	**Theresa:** "Multiply 729(3) and subtract 1. Then divide by 2."
Stella: $r=4$ "What is $5 + 20 + 80 + 320 + 1280 + 5120$?" 6825	**Theresa:** "I will have the answer if I multiply 5120(4), subtract 5, and then divide by 3."
Julian: $r=5$ "Let me see . . . How about $10 + 50 + 250 + 1250$?" 1560	**Theresa:** "No problem. Multiply 1250(5), subtract 10, and then divide by 4."
Henry: $r=-2$ "Hmmm . . . I bet I can stump you with $10 + (-20) + 40 + (-80) + 160$." 110	**Theresa:** "Pretty sneaky with the negatives, Henry, but the method still works. Multiply 160(−2) and subtract 10. This time divide by −3."

1. Verify that Theresa is correct for each series.

yes.

How can you tell all of the series are geometric?



2. What is Theresa's "trick"? Describe in words how to calculate the sum of any geometric sequence.

Multiply last term by common ratio, subtract the first term, and divide by one less than our common ratio (r-1).

$r = 2$ pg.597 in your book

3. Use Theresa's "trick" to calculate $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128$. Show all work and explain your reasoning.

$$\frac{128(2) - 1}{1} = \underline{255}$$

Theresa's "trick" really isn't a trick. It is known as Euclid's Method. An example of this method, along with a justification for each step, is shown.

Compute $\sum_{i=1}^{5} 3^{i-1} = 3^{1-1} + 3^{2-1} + 3^{3-1} + 3^{4+1} + 3^{5-1} = 1+3+9+27+81$

$S_5 = 1 + 3 + 9 + 27 + 81$

- The common ratio is 3.

$3S_5 = \quad 3 + 9 + 27 + 81 + 243$

$S_5 = 1 + 3 + 9 + 27 + 81$

$\times 3 \quad \times 3 \quad \times 3 \quad \times 3 \quad \times 3$

- Write $3S_n$ above the original series. Multiply each term of the original series by the common ratio. Line up each product above the original series.

$3S_5 = \quad 3 + 9 + 27 + 81 + 243$

$-S_5 = -(1 + 3 + 9 + 27 + 81)$

- Subtract to determine $3S_n - S_n = 2S_n$.

first term

$2S_5 = -1 + 243$ ← $81 \cdot 3$ last term $\cdot\, r$

$\frac{2S_5}{2} = \frac{242}{2}$

- Divide by 2.

$S_5 = 121$

one less than common ratio

In all of the examples, Theresa knew that she could calculate each sum by first multiplying the last term by the common ratio and subtracting the first term. Then she could divide that quantity by one less than the common ratio.

In other words, $S_n = \dfrac{\text{(Last Term)(Common Ratio)} - \text{(First Term)}}{\text{(Common Ratio} - 1)}$.

4. Analyze the worked example.

a. In the worked example, why multiply both sides of the equation by 3? Does the algorithm still work if you multiply by a different number? Explain your reasoning.

Common ratio; if you multiplied by a different number, you could get the sum, but you wouldn't be multiplying by the common ratio and dividing by $r-1$.

pg.598 in your book

pg.598 in your book
5 mins to finish #5 on pg.598

b. Why do you always divide by one less than the common ratio?

to get S_n by itself & solve for the sum

The formula to compute ~~any geometric series~~ becomes $\boxed{S_n = \frac{g_n(r) - g_1}{r - 1}}$ where g_n is the last term, r is the common ratio, and g_1 is the first term.

5. Apply Euclid's Method to compute each.

$g_1 = 1$
$r = 10$
$g_n = 1{,}000{,}000$

a. $1 + 10 + 100 + \cdots + 1{,}000{,}000$

$$S_n = \frac{1000000(10) - 1}{10 - 1} = 1{,}111{,}111$$

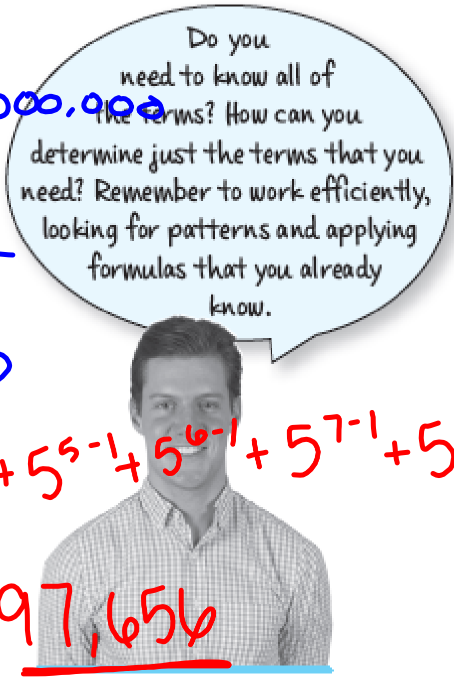
b. $10 + 20 + 40 + 80 + 160 + 320$

$r = 2$

$$S_6 = \frac{320(2) - 10}{2 - 1} = 630$$

c. $\sum_{k=1}^{8} 5^{k-1} = 5^{1-1} + 5^{2-1} + 5^{3-1} + 5^{4-1} + 5^{5-1} + 5^{6-1} + 5^{7-1} + 5^{8-1}$

$$S_8 = \frac{5^7(5) - 1}{5 - 1} = \frac{5^8 - 1}{4} = 97{,}656$$



d. A sequence with 9 terms, a common ratio of 2, and a first term of 3.
"n" "r" "g_1"

$g_9 = 3(2^{9-1}) = 768$ ← last term

$$S_9 = \frac{768(2) - 3}{2 - 1} = 1533$$

pg.598 in your book

Recall previously you used long division to determine each quotient:

Polynomial Long Division		**Rewritten Using the Reflexive and Commutative Properties of Equality**
Example 1 $\frac{r^3-1}{r-1} = r^2 + r + 1$	$\longrightarrow$	$1 + r + r^2 = \frac{r^3-1}{r-1}$
Example 2 $\frac{r^4-1}{r-1} = r^3 + r^2 + r + 1$	$\longrightarrow$	$1 + r + r^2 + r^3 = \frac{r^4-1}{r-1}$
Example 3 $\frac{r^5-1}{r-1} = r^4 + r^3 + r^2 + r + 1$	$\longrightarrow$	$1 + r + r^2 + r^3 + r^4 = \frac{r^5-1}{r-1}$

pg.599 in your book

Each Example represents a geometric series, where r is the common ratio and $g_1 = 1$. Each geometric series can be written in summation notation.

Example 1: $n = 3$ $\sum_{i=1}^{3} r^{i-1}$ or $\sum_{i=0}^{2} r^{i}$

Example 2: $n = 4$ $\sum_{i=1}^{4} r^{i-1}$ or $\sum_{i=0}^{3} r^{i}$

1. For each Example, explain why the power of the common ratio in the summation notation is different, yet still represents the series.

2. Identify the number of terms in the series in Example 3, and then write the series in summation notation.

3. Use the pattern generated from repeated polynomial long division to write a formula to compute any geometric series $1 + r + r^2 + r^3 + \cdots + r^{n-1}$ where n is the number of terms in the series, r is the common ratio, and $g_1 = 1$.

$$\sum_{i=0}^{n} r^i = \frac{r^n - 1}{r-1} = S_n$$

Jan 12-12:54 PM

pg.599 in your book

You can show a proof of $S_n = \frac{r^n - 1}{r - 1}$ where S_n is a series in the form $r^0 + r^1 + r^2 + r^3 + \cdots + r^{n-1}$ with n-terms and a common ratio r.

$$S_n = r^0 + r^1 + r^2 + r^3 + \cdots + r^{n-1}$$

$$rS_n = \quad r^1 + r^2 + r^3 + \cdots + r^{n-1} + r^n$$

x r x r x r x r x r

$$S_n = r^0 + r^1 + r^2 + \cdots + r^{n-2} + r^{n-1}$$

- Write rS_n above the original series. Multiply each term by r. Line up each product above the original series.

$$\begin{aligned} rS_n &= \quad \cancel{r^1} + \cancel{r^2} + \cancel{r^3} + \cdots + \cancel{r^{n-1}} + r^n \\ -\quad S_n &= -(1 + \cancel{r^1} + \cancel{r^2} + \cancel{r^3} + \cdots + \cancel{r^{n-1}}) \\ \hline rS_n - S_n &= -1 + r^n \\ S_n(r - 1) &= r^n - 1 \end{aligned}$$

- Subtract $rS_n - S_n$. Eliminate terms that subtract to 0.

$$\frac{S_n(r-1)}{(r-1)} = \frac{(r^n - 1)}{(r-1)}$$

- Divide by $(r - 1)$.

$$S_n = \frac{r^n - 1}{r - 1}$$

pg.600 in your book
take 5 mins to finish

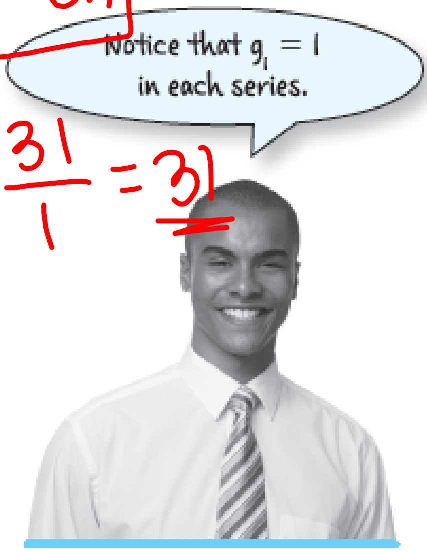
4. Identify the number of terms, the common ratio, and g_1 for each series. Then compute each.

$$\boxed{\frac{r^n-1}{r-1}=S_n}$$

a. $1 + 2^1 + 2^2 + 2^3 + 2^4$

$n=5$
$r=2$
$g_1=1$

$$S_5=\frac{2^5-1}{2-1}=\frac{31}{1}=31$$



b. $1 + 5 + 25 + 125 + 625$

c. $1 + (-2) + 4 + (-8) + 16 + (-32)$

pg.601 in your book

The formula to compute a geometric series that Perry used is $S_n = \frac{g_1(r^n - 1)}{r - 1}$.

Recall Euclid's Method to compute a geometric series is $S_n = \frac{g_n(r) - g_1}{r - 1}$.

equivalent

pg.602 in your book

7. Rewrite each series using summation notation.

a. 4 + 12 + 36 + 108 + 324

$r=3$ $g_1=4$ $g_n = g_1 \cdot r^{n-1}$

$g_n = 4 \cdot 3^{n-1}$

$\sum_{i=1}^{5} 4 \cdot 3^{i-1}$

b. 64 + 32 + 16 + 8 + 4 + 2 + 1

$r = \frac{1}{2}$ $g_1 = 64$

$g_n = 64 \cdot \left(\frac{1}{2}\right)^{n-1}$

$\sum_{i=1}^{7} 64\left(\frac{1}{2}\right)^{n-1}$

$S_n = \frac{r^{n}-1}{r-1}$

8. Compute each geometric series.

a. $\sum_{i=1}^{4} 6^{i-1}$

$g_1 = 1$ $n=4$

$r=6$

$S_4 = \frac{6^4 - 1}{6-1} = \frac{1295}{5} = \boxed{259}$

$6^{1-1} + 6^{2-1} + 6^{3-1} + 6^{4-1}$

$1 + 6 + 36 + 216$

b. $10\sum_{i=0}^{4} 3^i = \sum_{i=0}^{4} 10 \cdot 3^i$

c. $6\sum_{i=0}^{4} \left(\frac{1}{3}\right)^i$

NOT in your book

1. Two popular Florida tourist attractions have been competing for visitors since they each opened 15 years ago. Fantasy World had 100,000 visitors in their 1st year and their number of visitors increased by 2% each year over the 15-year period. Vacation Land had 90,000 visitors in their 1st year and their number of visitors increased by 4% each year over the same period.

   a. Determine which tourist attraction had the most visitors since the two attractions opened 15 years ago.

   b. Fantasy World has had an admission price of $20 per person since they opened. If the owners keep their price the same, they expect to maintain a 2% yearly increase in attendance. If they lower the price of admission to $18 per person, the owners expect their yearly attendance to increase by 3% each year beginning next year. Should the owners of Fantasy World lower the price of admission to $18 over the next 10 years? Explain your reasoning.

NOT in your book

2. Ryan and Morgan have competed in the Boston Marathon for 9 consecutive years. In Ryan's 1st year, he ran the marathon in 3.5 hours and he has steadily decreased his time by 3% each year. In Morgan's 1st year, he ran the marathon in 3.3 hours and he has steadily decreased his time by 2% each year.

   a. Which of the 2 runners had the fastest time in their 9th marathon? Round decimals to the nearest hundredth.

   b. Which of the 2 runners had the fastest total time if they combine each of their 9 marathon times? Round decimals to the nearest hundredth.

Jan 14-9:07 AM

Homework
Finish Lesson 8.3