Do you need chapter 8?

Questions on 8.1?

Answer the question below in your notes for review.

Several students collect a donation of 20 cans during the 1st day of their 10-day canned food drive. Their goal is to collect twice as many cans as they did the previous day for the remaining days of the food drive. Write an explicit formula to represent this situation and use the formula to calculate the amount of cans the students will collect on the final day of the drive if they meet their daily goals.

$r=2$

$g_1 = 20$

$g_2 = 40$ $\cdot 2$

$g_3 = 80$ $\cdot 2$

$g_4 = 160$

$\vdots$

$g_n = g_1 \cdot r^{n-1}$

$g_{10} = 20 \cdot 2^{10-1}$

$g_{10} = 20 \cdot 2^9$

$g_{10} = 10{,}240$ cans

Dec 10-3:22 PM

This Is Series(ous) Business

Finite Arithmetic Series

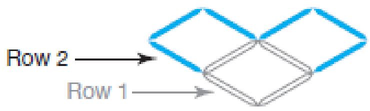
8.2

pg.581-582 in your book.

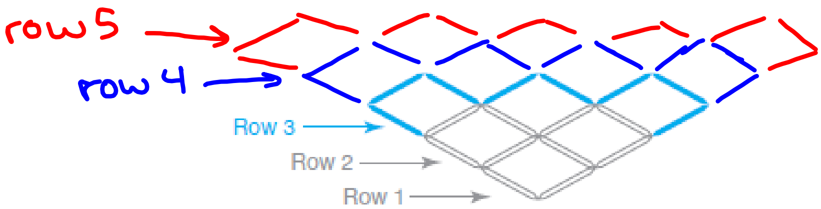
Josephine is helping her little brother Pauley with his latest art project. He is using toothpicks to create a *tessellation*. A **tessellation** is created when a geometric shape is repeated over a two-dimensional plane such that there are no overlaps and no gaps.

Pauley starts his tessellation project by gluing toothpicks to a large piece of poster board to make a single diamond shape. This is the first row.

Then, he places additional toothpicks parallel to the first row to create the second row. The second row consists of two diamond shapes.



He continues to place toothpicks in this manner, so that each row has one more diamond shape than the previous row. The first three rows of Pauley's tessellation are shown.



1. Sketch the next two rows of the tessellation on the previous diagram.

*Take 5 mins to finish pg.582-583

$a_1 = 4$

$d = 2$

$a_n = a_1 + d(n-1)$

$a_n = 4 + 2(n-1)$

$a_n = 4 + 2n - 2$

$\boxed{a_n = 2n + 2}$

Nov 19-8:31 AM

pg.584 in your book

You know how to determine the nth term of a sequence. However, sometimes it is necessary to determine the *sum* of the terms in a sequence.

A **series** is the sum of terms in a given sequence. The sum of the first n terms of a sequence is denoted by S_n. For example, S_3 is the sum of the first three terms of a sequence.

There is a special notation for the summation of terms using a capital sigma, Σ:

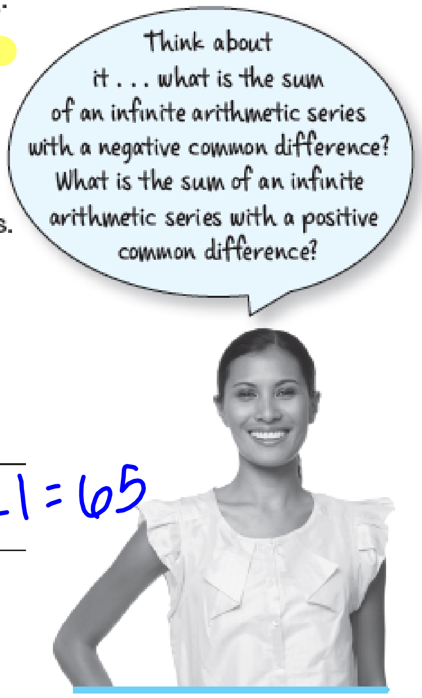
$$S_n = \sum_{i=1}^{n} a_i$$

upper bound of summation (n); an indexed variable representing each successive term in the series (a_i); index of summation (i); lower bound of summation (1)

This expression means sum the values of a, starting at a_1 and ending with a_n.

In other words, $S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$.

A series can be *finite* or *infinite*. A **finite series** is the sum of a finite number of terms. An **infinite series** is the sum of an infinite number of terms. For example, the sum of all of the even integers from 1 to 100 is a finite series, and the sum of all of the even whole numbers is an infinite series.



7. Use sigma notation to rewrite each finite series, and then compute.

a. 5 + 9 + 13 + 17 + 21

$S_2 = \sum_{i=1}^{2} a_i = 5+9=14$

$S_5 = \sum_{i=1}^{5} a_i = 5+9+13+17+21=65$

b. 3 + 6 + 12 + 24 + 48 + 96 + 192

$S_1 = \sum_{i=1}^{1} a_i = 3$

$S_7 = \sum_{i=1}^{7} a_i = 3+6+12+24+48+96+192=381$

8. Use sigma notation to represent the total number of toothpicks Pauley needs to complete 5 rows of his tessellation. Then, use your table in Question 2 to calculate this amount.

$$\sum_{i=1}^{5} a_i = 4+6+8+10+12=40$$

pg.585 in your book

Remember that an arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant. An arithmetic series is the sum of an arithmetic sequence.

You can compute a finite arithmetic series by adding each individual term, but this can take a lot of time. A famous mathematician named Carl Friedrich Gauss developed another way to compute a finite arithmetic series.

As the story goes, when Gauss was in elementary school, his teacher asked the class to calculate the sum of the first 100 positive integers. Apparently, Gauss determined the answer in a matter of seconds! How did Gauss determine his answer so quickly?

1. Complete the steps and answer the questions to see how Gauss was able to calculate the sum of the first 100 positive integers so quickly.

   a. The series S_{100} is shown. The same series in descending order is shown beneath it. Add the series by computing the sum of each pair of vertical, or partial sums.

$$S_{100} = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$
$$+S_{100} = 100 + 99 + 98 + \cdots + 3 + 2 + 1$$
$$2S_{100} = 101 + 101 + 101 + \cdots + 101 + 101 + 101$$

   b. What do you notice about each partial sum?

   101

   c. How many partial sums are there in this series?

   100

   d. Write the sum of the partial sums.

   $2S_{100} = 100(101)$

   e. To arrive at the total in part (d), you actually added each term of the series twice. How could you calculate the correct total from the sum of the partial sums, or S_{100}?

   divide by 2

   f. What is S_{100}?

   $S_{100} = \frac{100(101)}{2} = 5050$

$\frac{100(101)}{2} = 50(101)$

101

$S_{100} = 1+2+3+4+5+\cdots+95+96+97+98+99+100$

101 101 101

$50(101) = 5050$

pg.586 in your book

Gauss's method can be generalized for any finite arithmetic series.

2. Consider a finite arithmetic series S_n written as a sum of its terms.

$$S_n = a_1 + a_2 + a_3 + \cdots + a_{n-2} + a_{n-1} + a_n$$

Complete the steps shown to determine Gauss's formula to compute any finite arithmetic series.

a. First, write S_n in terms of a_1, a_n, and the common difference d. Remember that for an arithmetic sequence, $a_n = a_1 + d(n - 1)$.

$$S_n = a_1 + (a_1+d) + (a_1 + 2d) + \cdots + (a_n - 2d) + (a_n-d) + a_n$$

b. Then, write S_n in reverse order.

$$S_n = a_n + (a_n - d) + (a_n-2d) + \cdots + (a_1 + 2d) + (a_1+d) + a_1$$

c. Add the series, keeping the "+" and "=" signs vertically aligned.

$$\begin{aligned} S_n &= a_1 + (a_1+d) + (a_1+2d) + \cdots + a_n-2d + a_n-d + a_n \\ +S_n &= a_n + a_n-d + a_n-2d + \cdots + a_1+2d + a_1+d + a_1 \\ \hline 2S_n &= a_n+a_1 + a_n+a_1 + a_n+a_1 + \cdots + a_n+a_1 + a_n+a_1 + a_n+a_1 \end{aligned}$$

d. Identify each partial sum.

$a_n + a_1$

e. Fill in the blanks to show the sum of the partial sums.

$$2S_n = n(a_1 + a_n)$$

f. Fill in the blanks to write the formula for S_n.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

g. Describe Gauss' rule to compute any finite arithmetic series by completing the sentence.

Add the first term and the last term of the series, multiply the sum by the number of terms of the series, and divide by 2.

pg.587 in your book

So, Gauss's formula to compute the first ~~n terms of an~~ arithmetic series is shown.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

*Finish pgs.587-593 for HW

3a) $S_5 = \frac{5(a_1+a_5)}{2} = \frac{5(4+12)}{2} = \frac{5(16)}{2} = \underline{\underline{40}}$

3b) $a_{18} = 38$

$S_{18} = \frac{18(a_1+a_{18})}{2} = \frac{18(4+38)}{2}$

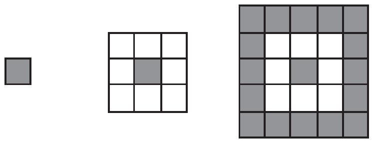
$= \underline{\underline{378}}$

Not enough for 18 rows.

Jan 12-12:54 PM

NOT in your book

1. Jill is laying 1 foot by 1 foot tiles that each have 1 gray side and 1 white side in a room that measure 25 feet by 25 feet. She lays a gray tile in the center of the room. Next, she lays a ring of white tiles around the center tile. Then, she lays a ring of gray tiles around the white tiles and continues the pattern in this manner. The first 3 steps in her pattern are shown.



   a. Determine the pattern in the number of tiles added in each ring.

   b. Write an explicit formula to represent the number of tiles added in ring *n*.

   c. Determine the number of tile rings that must be added around the center tile to completely fill the room's floor. Explain your reasoning.

   d. Determine the number of tiles needed to completely cover the floor. Explain your reasoning.

   e. Jill only has enough money to buy 400 tiles. She decides to lay as many complete rings around the center tile as she can. How many complete tile rings can Jill lay with 400 tiles? Of the 400 tiles, how many tiles will Jill use if she only lays complete rings?

Homework

Finish Lesson 8.2