# Starter

Do you remember how to rationalize the denominator? Work on these two problems with your group, you may **not** have a radical sign in the denominator!

$$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$$

$$\frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{\sqrt{49}} = \frac{4\sqrt{7}}{7}$$

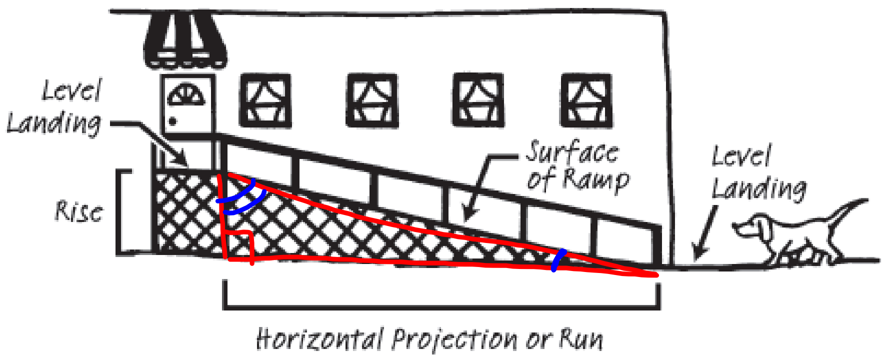
Perfect Square: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ....

# 8.2 The Tangent Ratio

## Tangent Ratio, Cotangent Ratio, and Inverse Tangent

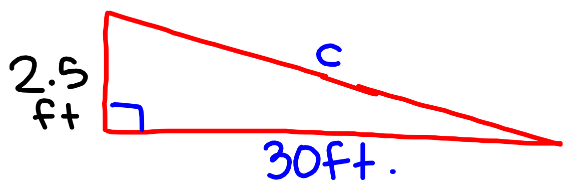
PG.579-580 IN YOUR BOOK

The maximum incline for a safe wheelchair ramp should not exceed a ratio of 1 : 12. This means that every 1 unit of vertical rise requires 12 units of horizontal run. The maximum rise for any run is 30 inches. The ability to manage the incline of the ramp is related to both its steepness and its length.




Troy decides to build 2 ramps, each with the ratio 1 : 12.

1. The first ramp extends from the front yard to the front porch. The vertical rise from the yard to the porch is 2.5 feet.
   a. Draw a diagram of the ramp. Include the measurements for the vertical rise and horizontal run of the ramp.




$\frac{\text{rise}}{\text{run}} \quad \frac{1}{12} = \frac{2.5}{x}$

$12(2.5) = x$

$30 \text{ ft} = x$

   b. Calculate the length of the surface of the ramp.

$(2.5)^2 + (30)^2 = c^2$

$6.25 + 900 = c^2$

$\sqrt{906.25} = \sqrt{c^2}$

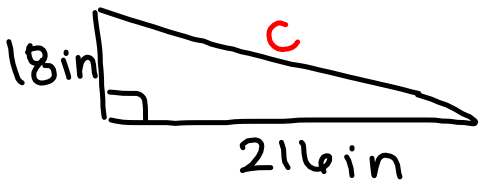
$\boxed{30.1 \text{ ft} = c}$

Sep 14-9:16 AM

PG.581 IN YOUR BOOK

2. The second ramp extends from the deck on the back of the house to the backyard. The vertical rise from the yard to the deck is 18 inches.

   a. Draw a diagram of the ramp. Include the measurements for the vertical rise and horizontal run of the ramp.




$$\frac{1}{12} = \frac{18}{x}$$

$$18 \cdot 12 = x$$

$$216\text{in} = x$$

   b. Calculate the length of the surface of the ramp.

$$18^2 + 216^2 = c^2$$

$$324 + 46656 = c^2$$

$$\sqrt{46980} = \sqrt{c^2}$$

$$216.75 = c$$

in

3. Compare the two ramps. Are the triangles similar? Explain your reasoning.

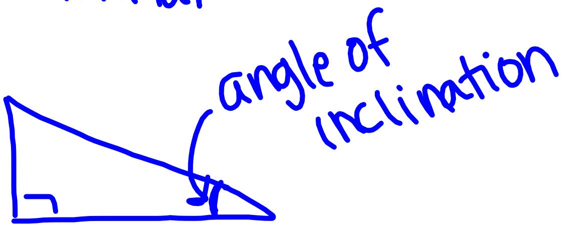
$$\frac{2.5}{18} = \frac{30}{216}$$

$$0.138 = 0.138$$

Yes, SAS ~ with the 90° ∠ and the two sides that include that 90° ∠.

4. Compare and describe the angles of inclination of the two ramps.

The angle of incline is the same because our triangles are similar.



# PG.582 IN YOUR BOOK

In the wheelchair ramp problem, Troy used 1 : 12 as the ratio of the rise of each ramp to the run of each ramp.

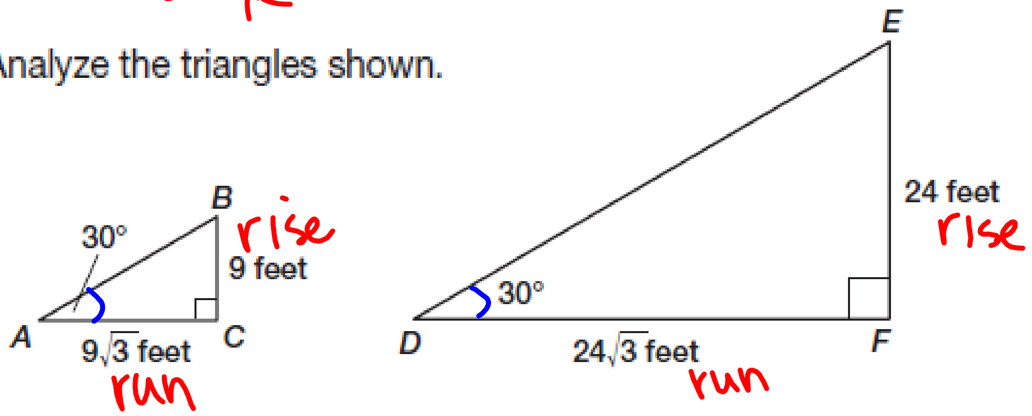
**1.** Describe the shape of each wheelchair ramp.

Right $\triangle$

**2.** What does the ratio of the rise of the ramp to the run of the ramp represent?

Slope

**3.** Analyze the triangles shown.




**a.** Verify the triangles are similar. Explain your reasoning.

Yes, AA similarity.

**b.** Calculate the ratio of the rise to the run for each triangle. How do the ratios compare?

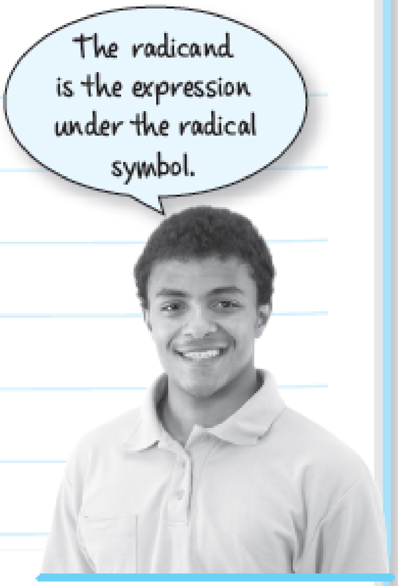
$$\frac{\cancel{9}^{1}}{\cancel{9}_{1}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{\cancel{24}^{1}}{\cancel{24}_{1}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

# PG.583 IN YOUR BOOK

A standard mathematical convention is to write fractions so that there are no irrational numbers in the denominator. **Rationalizing the denominator** is the process of rewriting a fraction so that no irrational numbers are in the denominator.

To rationalize the denominator of a fraction involving radicals, multiply the fraction by a form of 1 so that the product in the denominator includes a perfect square radicand. Then simplify, if possible.




Example 1:

$$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{4}}$$
$$= \frac{10\sqrt{2}}{2}$$
$$= 5\sqrt{2}$$

Example 2:

$$\frac{3}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{5\sqrt{9}}$$
$$= \frac{3\sqrt{3}}{5 \cdot 3}$$
$$= \frac{3\sqrt{3}}{15}$$
$$= \frac{\sqrt{3}}{5}$$

**4.** Rewrite your answers in Question 3, part (b), by rationalizing the denominators. Show your work.

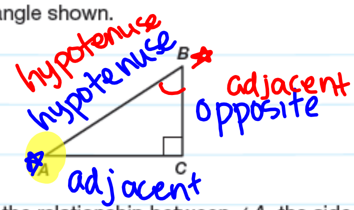
$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \boxed{\frac{\sqrt{3}}{3}}$$

on pg.584, work on #5-6 in your book

## PG.584 IN YOUR BOOK

The **tangent (tan)** of an acute angle in a right triangle is the ratio of the length of the side that is opposite the reference angle to the length of the side that is adjacent to the reference angle. The expression "tan *A*" means "the tangent of ∠*A*."

Consider ∠*A* in the right triangle shown.




The tangent ratio describes the relationship between ∠*A*, the side opposite ∠*A*, and the side adjacent to ∠*A*.

opposite

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} = \frac{BC}{AC}$$

## PG.585 IN YOUR BOOK

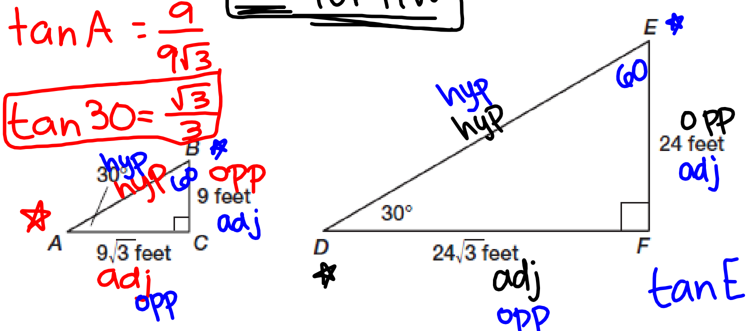
**7.** Complete the ratio that represents the tangent of ∠*B*.

$$\tan B = \frac{\text{length of side opposite } \angle B}{\text{length of side adjacent to } \angle B} = \frac{\boxed{AC}}{\boxed{BC}}$$

**8.** Determine the tangent values of all the acute angles in the right triangles from Questions 3 and 5. for HW

$\tan A = \frac{9}{9\sqrt{3}}$

$\boxed{\tan 30 = \frac{\sqrt{3}}{3}}$




$\tan B = \frac{{}^{1}\cancel{9}\sqrt{3}}{\cancel{9}_{1}}$

$\boxed{\tan 60 = \sqrt{3}}$

$\tan D = \frac{24}{24\sqrt{3}}$

$\boxed{\tan 30 = \frac{\sqrt{3}}{3}}$

$\tan E = \frac{\cancel{24}\sqrt{3}}{\cancel{24}}$

$\boxed{\tan 60 = \sqrt{3}}$

**9.** What can you conclude about the tangent values of congruent angles in similar triangles?

the tangent is the same

($\tan 60 = \sqrt{3}$)

**10.** Consider the tangent values in Question 8. In each triangle, compare tan 30° to tan 60°. What do you notice? Why do you think this happens?

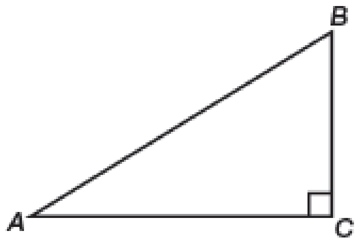
they're reciprocals

#11-14 on pgs.586-587 are **homework**
& **SKIP** problem 3 on pgs. 588-589

Dec 1-8:48 AM

PG.589 IN YOUR BOOK

The **cotangent (cot)** of an acute angle in a right triangle is the ratio of the length of the side that is adjacent to the angle to the length of the side that is opposite the angle. The expression "cot *A*" means "the cotangent of $\angle A$."




1. Complete the ratio that represents the cotangent of $\angle A$.

$$\cot A = \frac{\text{length of side adjacent to } \angle A}{\text{length of side opposite } \angle A} = \frac{\boxed{\phantom{xx}}}{\boxed{\phantom{xx}}}$$

2. Prove algebraically that the cotangent of $A = \frac{1}{\tan A}$.

pg.590 in your book

**4.** A ski slope at Snowy Valley has an average angle of elevation of 21°.


**a.** Calculate the vertical height of the ski slope *x* using the cotangent ratio.

**b.** Calculate the vertical height of the ski slope *x* using the tangent ratio.

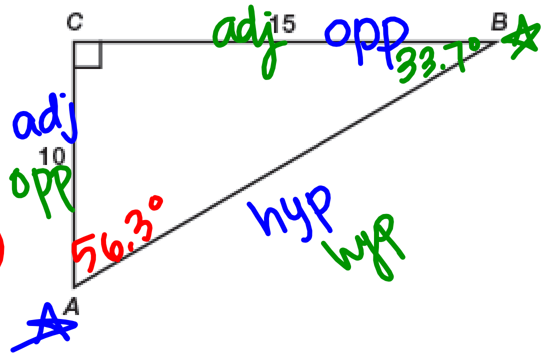
Nov 19-2:51 PM

pg.591 in your book

The **inverse tangent** (or arc tangent) of $x$ is defined as the measure of an acute angle whose tangent is $x$. If you know the length of any two sides of a right triangle, it is possible to compute the measure of either acute angle by using the inverse tangent, or the $\boxed{\tan^{-1}}$ button on a graphing calculator.

inverse tangent
$\tan^{-1}$

In right triangle $ABC$, if $\tan A = x$, then $\tan^{-1} x = m\angle A$.




1. Consider triangle $ABC$ shown.

   a. If $\tan A = \frac{15}{10}$, then calculate $\tan^{-1}\left(\frac{15}{10}\right)$ to determine $m\angle A$.

$$\tan A = \frac{15}{10}$$
$$\cancel{\tan^{-1}}(\cancel{\tan} A) = \tan^{-1}\left(\frac{15}{10}\right)$$
$$A = \tan^{-1}\left(\frac{15}{10}\right)$$
$$\boxed{A = 56.3°}$$

   b. Determine the ratio for $\tan B$, and then use $\tan^{-1}(\tan B)$ to calculate $m\angle B$.

$$\tan B = \frac{10}{15}$$
$$\cancel{\tan^{-1}}(\cancel{\tan} B) = \tan^{-1}\left(\frac{10}{15}\right)$$
$$B = \tan^{-1}\left(\frac{10}{15}\right)$$
$$\boxed{B = 33.7°}$$

pg.592 in your book

   c. Add $m\angle A$ and $m\angle B$. Does your sum make sense in terms of the angle measures of a triangle?

Yes, $33.7 + 56.3 = 90$; it makes sense. Our $\triangle$ interior $\angle$s add to 180°

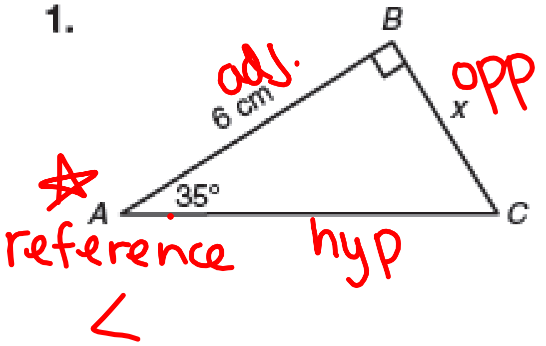
#2-3 on pg.592 in your book is **homework** &
problem 6 on pg.593 is also **homework**

not in your book

we used the inverse tangent to find ∠ measures.

Use the tangent ratio, the cotangent ratio, or the inverse tangent to solve for *x*. Round each answer to the nearest tenth.

**1.**

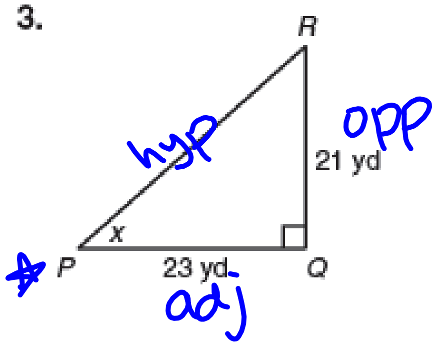



$$6 \cdot \frac{\tan 35}{1} = \frac{x}{6} \cdot 6$$

$$x = 6\tan 35$$

$$x = 4.2\text{ cm}$$

**3.**



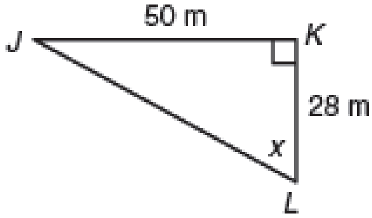

$$\tan x = \frac{21}{23}$$

$$\cancel{\tan^{-1}(\tan} x) = \tan^{-1}\left(\frac{21}{23}\right)$$

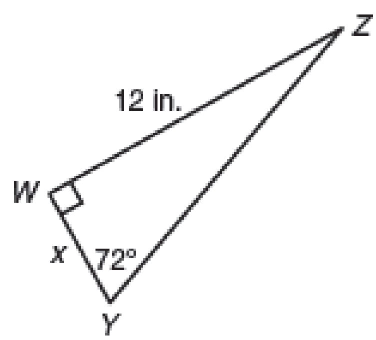
$$x = \tan^{-1}\left(\frac{21}{23}\right)$$

$$x = 42.4°$$

**4.**

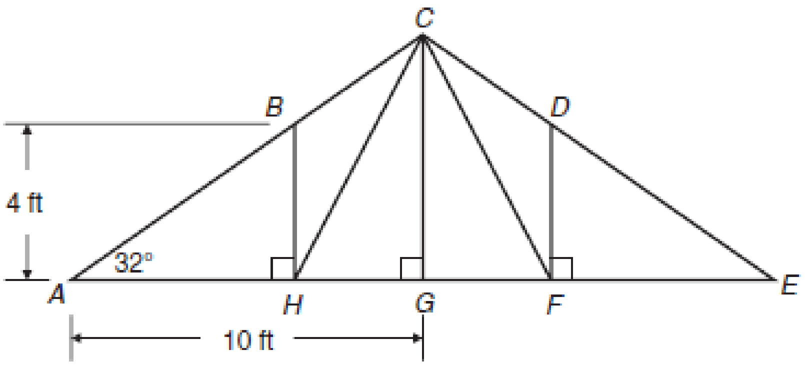



**2.**



not in your book

5. A roof truss is shown. Use the figure to complete parts (a) through (d). Round each answer to the nearest hundredth.




a. Determine the height *CG* of the roof truss.

b. Determine *AH*.

c. Determine the measure of angle *HCG*.

d. Determine the length *CH* of the support beam.

**Homework**

Finish Lesson 8.2