15

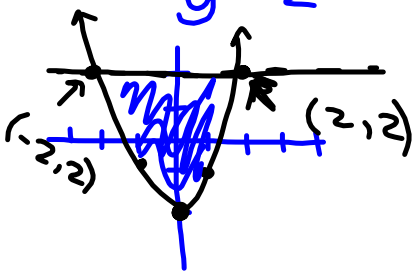
Questions on 8.1 HW?

(6) $A = \int_{-1}^{1} (x^2 + 2x^4)\,dx = \left[\frac{x^3}{3} + \frac{2x^5}{5}\right]_{-1}^{1} =$

$\left(\frac{1}{3} + \frac{2}{5}\right) - \left(-\frac{1}{3} - \frac{2}{5}\right) = \frac{11}{15} + \frac{11}{15} = \frac{22}{15}$ units^2

(15) $y = x^2 - 2$

$y = 2$



$A = \int_{-2}^{2} [2 - (x^2 - 2)]\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx =$

$= \left[-\frac{x^3}{3} + 4x\right]_{-2}^{2} = \left(-\frac{8}{3} + 8\right) - \left(\frac{8}{3} - 8\right)$

$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$ units^2

Nov 3-2:38 PM

8.2 Position, Velocity, Acceleration

What is the relationship between the three?

derivatives

position / displacement $x(t)$

↓

velocity $v(t) = x'(t)$

↓

acceleration $a(t) = v'(t) = x''(t)$

Integrals:

acceleration $a(t)$

↓ $v(t) = \int a(t)\,dt$

velocity

↓ $x(t) = \int v(t)\,dt = \iint a(t)\,dt$

position / displacement

Nov 14-10:50 AM

Relationship between function, 1st derivative, and 2nd derivative as seen on graph.

→ 1st derivative: where = 0, horizontal tangent line - max or min
where (+), function is increasing
where (−), function is decreasing

→ 2nd derivative: where (+), function is concave ↑;
where (−), function is concave ↓;
where = 0, point of inflection is possible.

Relationship between velocity, acceleration, and speed:

speed is the absolute value of velocity

position:

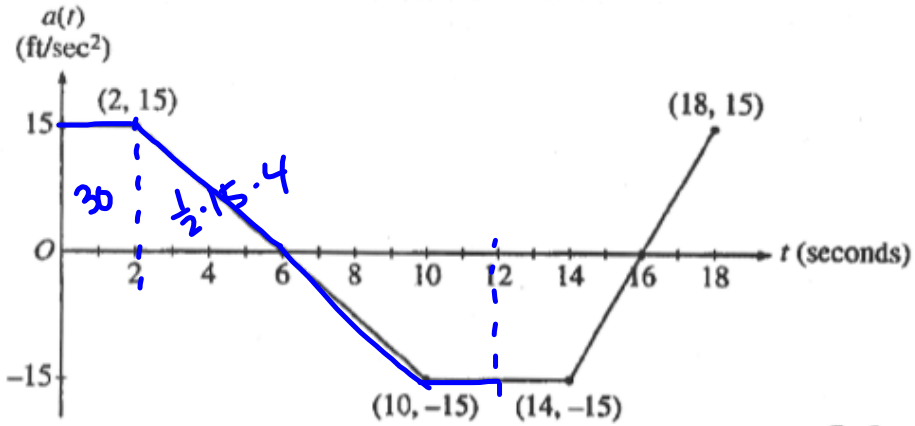
velocity: ft/sec → speed: ft/sec

acceleration: ft/sec^2

Example

(No calculator)

2001



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what tim does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

a) $a(2) = v'(2) = 15$, $15 > 0$ so yes, velocity is increasing at $t = 2$.

b) $t = 12$ $\quad v(12) - v(0) = \int_0^{12} a(t)\,dt = 0$

c) $t = 6$ (or endpoint)

$v(6) = 55 + \int_0^6 a(t)\,dt = 55 + 2 \cdot 15 + \frac{1}{2} \cdot 15 \cdot 4$

$= 115$ ft/sec

d) The car's velocity will not ever equal 0; the absolute minimum occurs at $t = 16$, where $f''(x)$ changes from − to + and

$v(16) = 115 + \frac{1}{2}(-15)(14) = 115 - 105 = 10$ ft/sec

Example

(Calculator allowed)

2011

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{\frac{t}{4}}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{\frac{t}{4}}\cos\left(e^{\frac{t}{4}}\right)$, and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

$v(5.5) = 2\sin(e^{5.5/4}) + 1 = -0.45337$

$a(5.5) = \frac{1}{2}e^{5.5/4}\cdot\cos(e^{5.5/4}) = -1.35857$

Speed is increasing at $t = 5.5$ b/c v & a have same sign.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$

$$\frac{1}{6-0}\int_0^6 v(t)\,dt = \frac{1}{6}\cdot 11.6963 = 1.95$$

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

$$\int_0^6 |v(t)|\,dt = 12.573$$

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

graph $v(t)$

↓

$v(t)$ changes from + to − at $t = 5.2$

$v(5.2) = 0$

$x(0) = 2$

$$x(5.2) = 2 + \int_0^{5.2} v(t)\,dt = 14.1348$$

Homework

8.2 WKS

Oct 21-8:47 AM