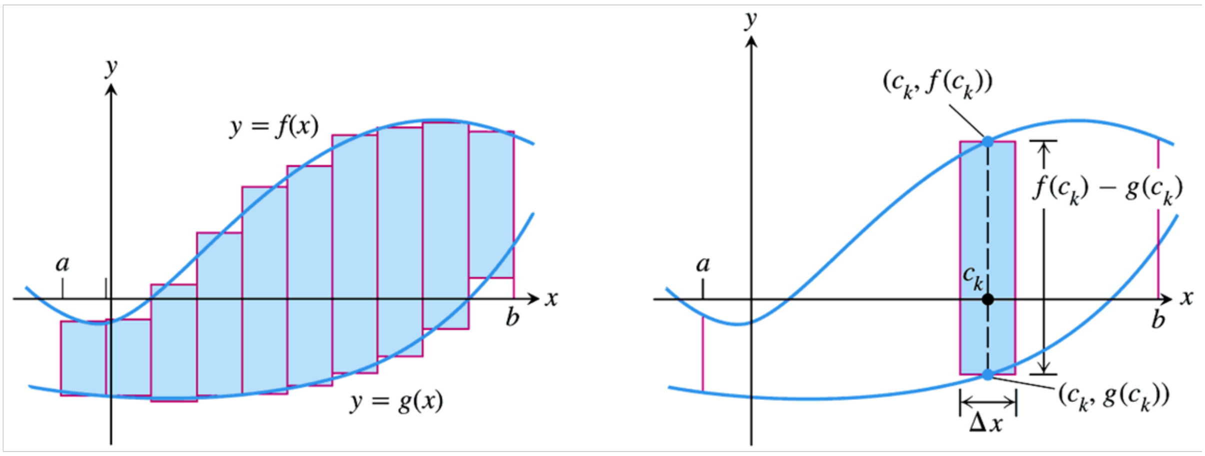
If you haven't checked off your unit 7 homework, get that ready to be checked off! We will start unit 8 today! :)

Also, turn in your Unit 7 Review if you haven't already

8.1 Areas in the Plane (8.2 in book)

Area Between Curves

Partition the region into vertical strips of equal width Δx.
Each rectangle has area $\left[f\left(c_k\right)-g\left(c_k\right)\right]\Delta x$ for some c_k in its respective subinterval. Approximate the area of each region with the Riemann sum $\sum\left[f\left(c_k\right)-g\left(c_k\right)\right]\Delta x$.



The limit of these sums as $\Delta x \to 0$ is $\int_a^b\left[f\left(x\right)-g\left(x\right)\right]dx$.

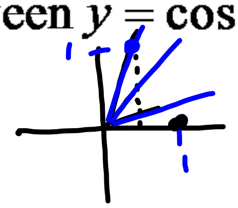
If f and g are continuous with $f(x) \geq g(x)$ throughout $\left[a,b\right]$, then the area between the curves $y=f(x)$ and $y=g(x)$ from a to b is the integral of $\left[f-g\right]$ from a to b,

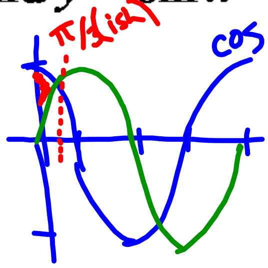
$$A=\int_a^b\left[f(x)-g(x)\right]dx$$

Example

$f(x)$ $g(x)$

Find the area of the region between $y = \cos x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{3}$.





$$A = \int_0^{\pi/3} (\cos x - \sin x)\,dx = \left[\sin x + \cos x\right]_0^{\pi/3} = \left[\left(\sin \tfrac{\pi}{3} + \cos \tfrac{\pi}{3}\right) - \left(\sin 0 + \cos 0\right)\right]$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) - (0+1) = \frac{\sqrt{3}+1}{2} - \frac{2}{2}$$

$$= \frac{\sqrt{3}+1-2}{2} = \boxed{\frac{\sqrt{3}-1}{2} \text{ units}^2}$$

Answer

Note that $y = \cos x$ is above $y = \sin x$ in the given interval.

$$A = \int_0^{\frac{\pi}{3}} [\cos x - \sin x]\,dx$$

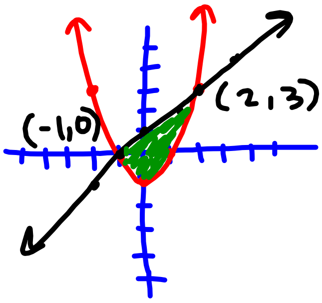
$$= [\sin x + \cos x]_0^{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}-1}{2} \text{ units squared.}$$

Example

Find the area of the region enclosed by the parabola $y = x^2 - 1$ and $y = x + 1$.

Graph the curves to view the region.



Intersection Points Algebraically:

$x^2 - 1 = x + 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

$A = \int_{-1}^{2} [(x+1) - (x^2-1)]\,dx =$

$\int_{-1}^{2} (-x^2 + x + 2)\,dx =$

$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2} =$

$\left[\left(-\frac{8}{3} + 2 + 4\right) - \left(- -\frac{1}{3} + \frac{1}{2} - 2\right)\right]$

$\frac{10}{3} - -\frac{7}{6} = \boxed{\frac{9}{2} \text{ units}^2}$

Answer

The limits of integration are found by solving the equation $x^2 - 1 = x + 1$.

The solutions are $x = -1$ and $x = 2$.

Since the line lies above the parabola on $[-1, 2]$, the area integrand is

$x + 1 - (x^2 - 1)$.

$$A = \int_{-1}^{2} \left[-x^2 + x + 2\right] dx$$

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$$

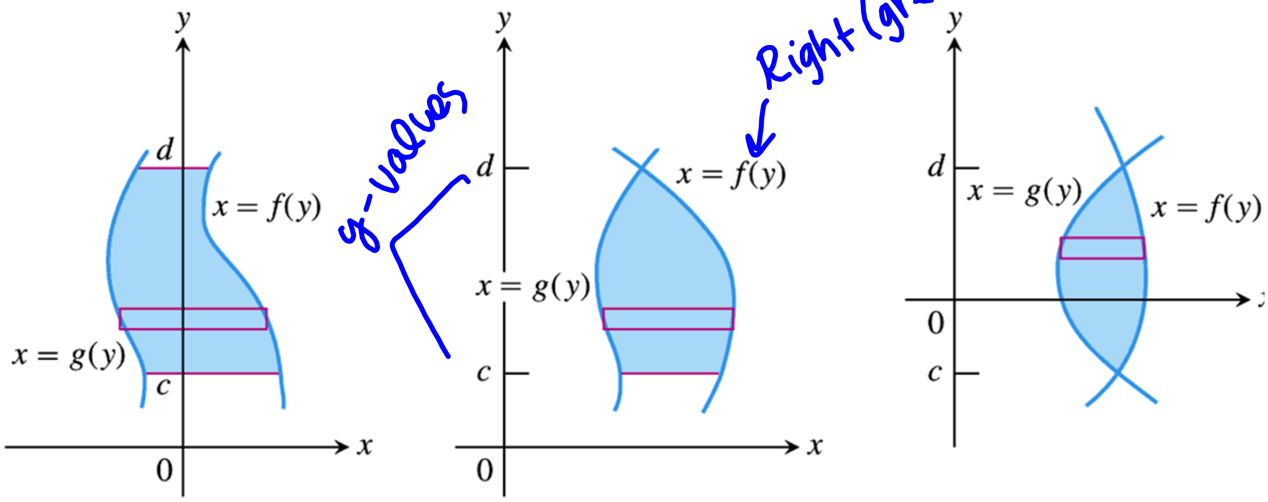
$$= \frac{9}{2} \text{ units squared.}$$

Integrating with Respect to y

If the boundaries of a region are more easily described by functions of y, use horizontal approximating rectangles.

Right (greater x-values)

For regions like these



use this formula

$$\boxed{A = \int_c^d [f(y) - g(y)]\, dy.}$$

Example

Find the area of the region bounded by the curves $x = y^2 - 1$ and $y = x + 1$.

Find intersection points algebraically:

$y^2 - 1 = y - 1$

$y^2 - y = 0$

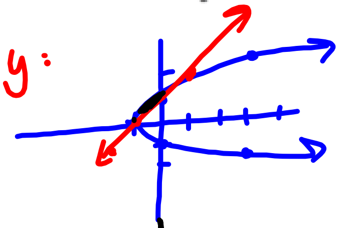
$y(y-1) = 0$

$y = 0, 1$

$0 - 1 = x$ $\quad$ $1 - 1 = x$

$-1 = x$ $\quad$ $0 = x$

$(-1, 0)$ $\quad$ $(0, 1)$



$x + 1 = y^2$

$\pm\sqrt{x+1} = y$

solve for x:

$y - 1 = x$

$$A = \int_0^1 \left[(y-1) - (y^2 - 1)\right] dy =$$

$$\int_0^1 (-y^2 + y)\,dy = \left[-\frac{y^3}{3} + \frac{y^2}{2}\right]_0^1 =$$

$$\left(-\frac{1}{3} + \frac{1}{2}\right) - (0) = \boxed{\frac{1}{6}\text{ units}^2}$$

Answer

Solve for x in terms of y: $x = y^2 - 1$ and $x = y - 1$. Find the points of intersection: $(-1,0)$ and $(0,1)$.

$$A = \int_0^1 \left[(y-1) - (y^2 - 1)\right] dy$$

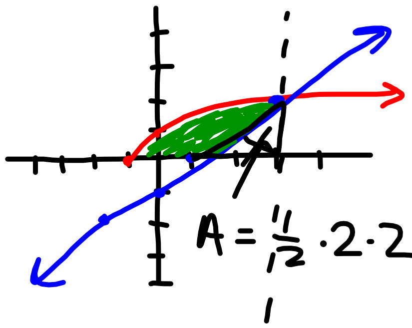
$$= \int_0^1 \left[y - y^2\right] dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$$

$$= \frac{1}{6}$$

Example

Find the area of the region enclosed by the graphs of $y=\sqrt{x+1}$, $y=x-1$ and the x-axis.



$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$$\int_{-1}^{3}\sqrt{x+1}\,dx - 2 =$$

$u = x+1$
$du = dx$

$$\int_{\cancel{-1}\;0}^{\cancel{3}\;4} u^{1/2}\,du - 2 = \left[\frac{(x+1)^{3/2}}{3/2}\right]_{-1}^{3} - 2$$

$$= \frac{2}{3}\left[(x+1)^{3/2}\right]_{-1}^{3} - 2$$

$4^{3/2} = \sqrt{4^3}$

$= \sqrt{4}^{\,3}$

$$= \frac{2}{3}\left((4)^{3/2} - 0^{3/2}\right) - 2$$

$$= \frac{2}{3}(8) - 2 = \frac{16}{3} - 2 = \boxed{\frac{10}{3}\text{ units}^2}$$

Answer

Find the area under the curve $y=\sqrt{x+1}$ over the interval $[-1,3]$ and subtract the area of the triangle:

$$A = \int_{-1}^{3}\left(\sqrt{x+1}\right)dx - \frac{1}{2}(2)(2)$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}}\Big]_{-1}^{3} - 2$$

$$= \frac{10}{3} \text{ units squared}$$

Dec 15-9:06 AM

Homework

8.1: pg.399-400 #3-33 (X3)

SKIP 21