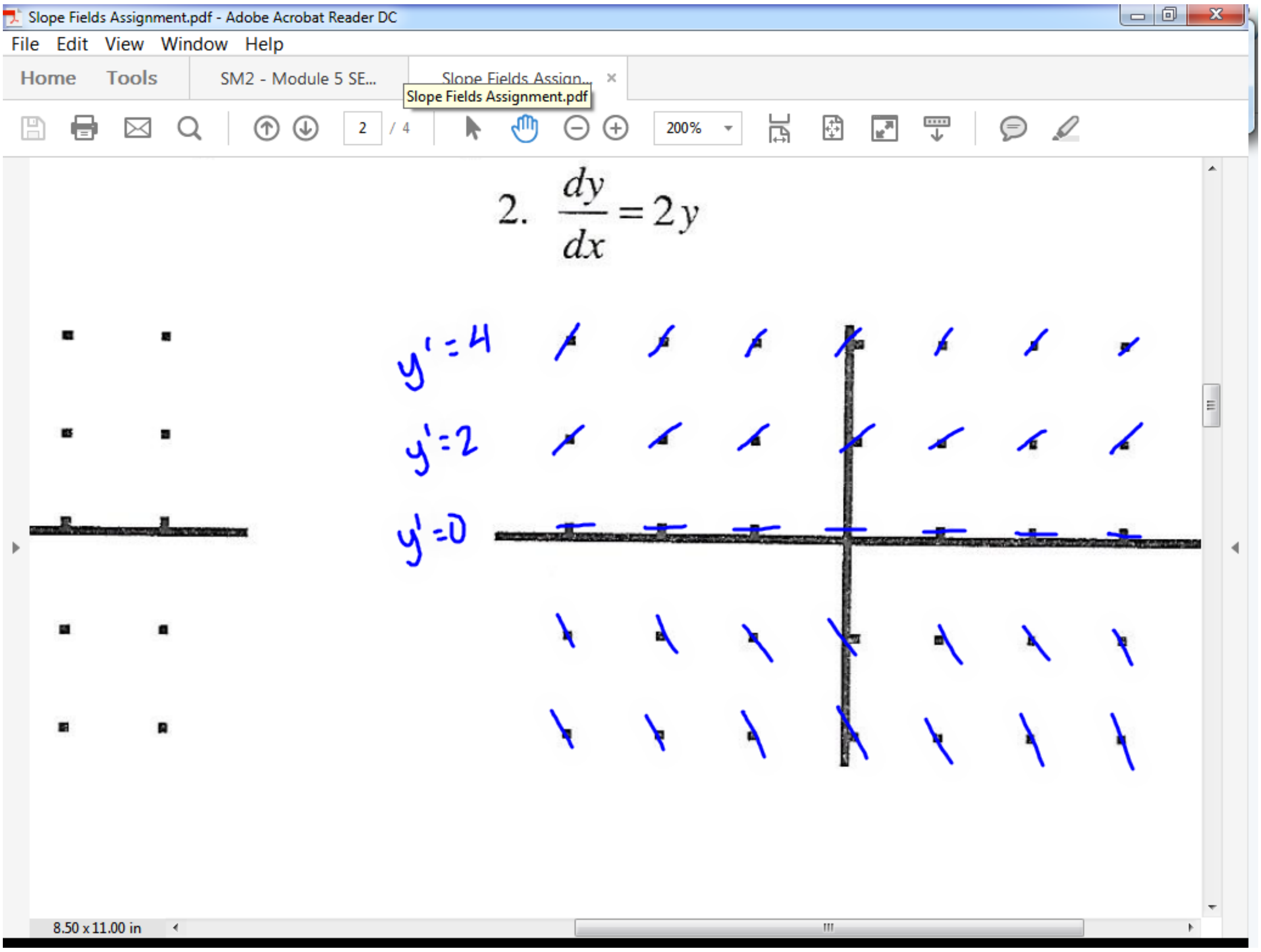
Questions on Slope Fields Packet?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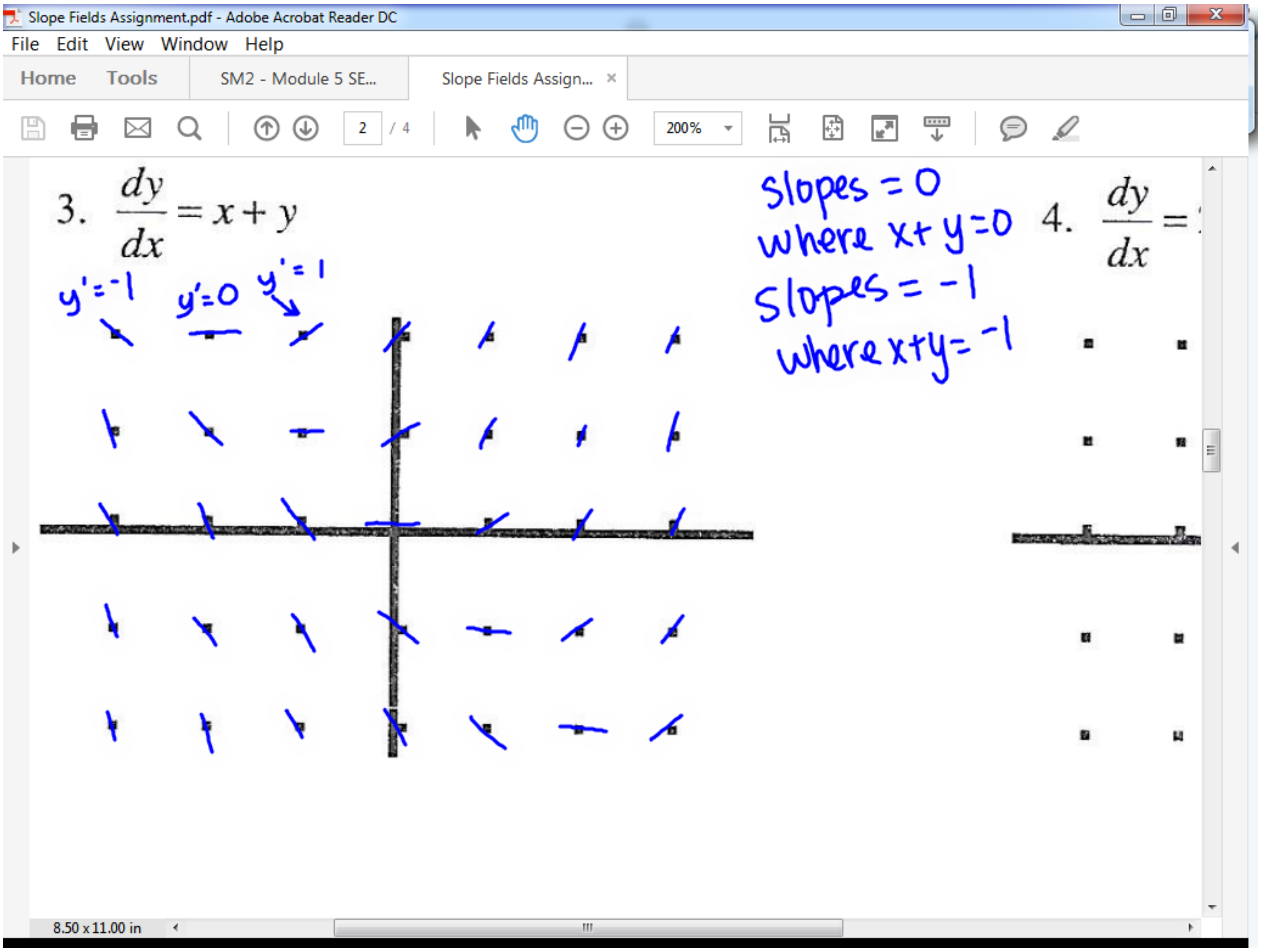
2. $\frac{dy}{dx} = 2y$
y' = 4
y' = 2
y' = 0
8.50 x 11.00 in

Feb 9-9:23 AM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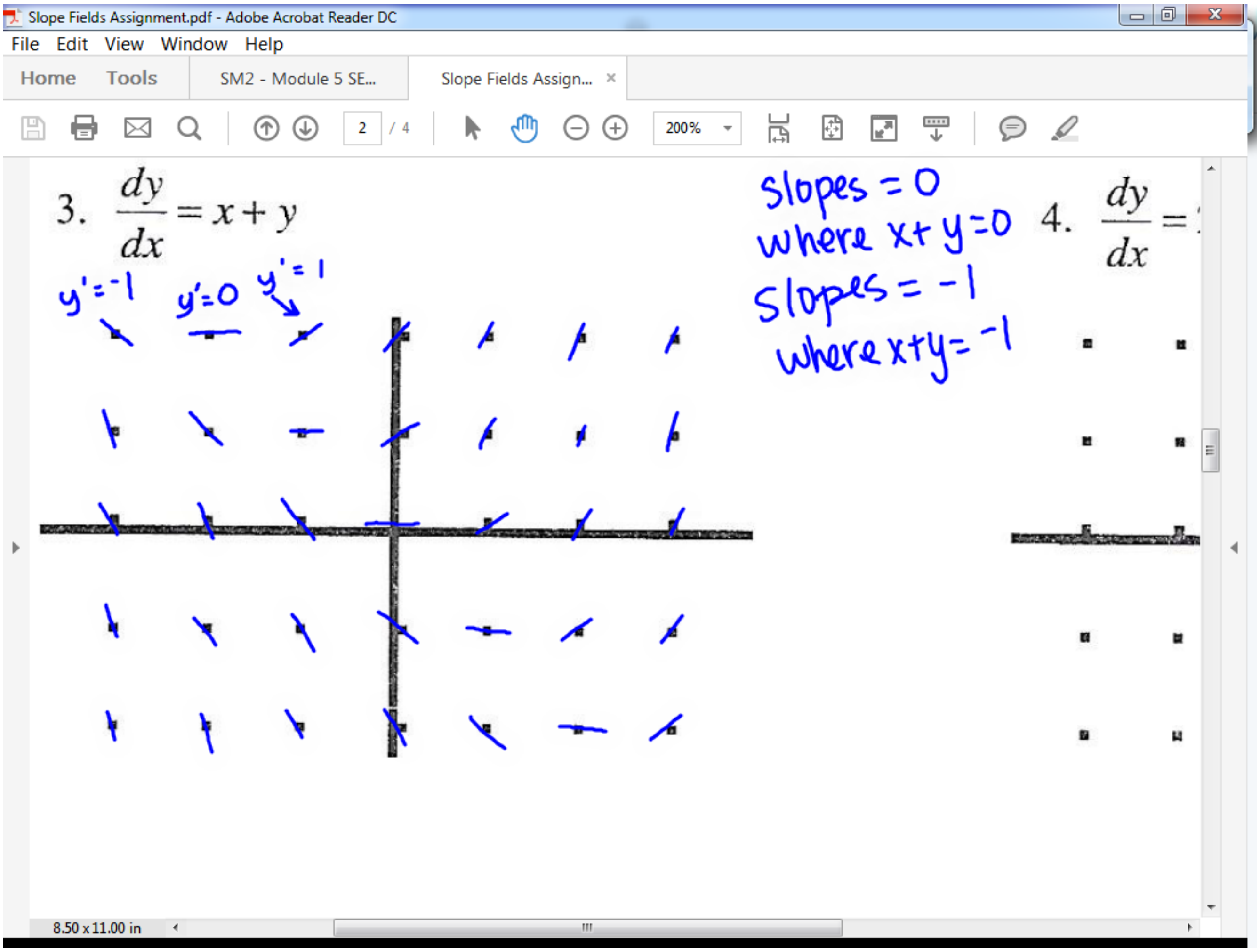
3. $\frac{dy}{dx} = x + y$
y' = -1
y' = 0
y' = 1
slopes = 0
where x + y = 0
slopes = -1
where x + y = -1
4. $\frac{dy}{dx} =$
8.50 x 11.00 in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3. $\frac{dy}{dx} = x + y$
$y' = -1$
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$y' = 1$
slopes = 0
where $x + y = 0$
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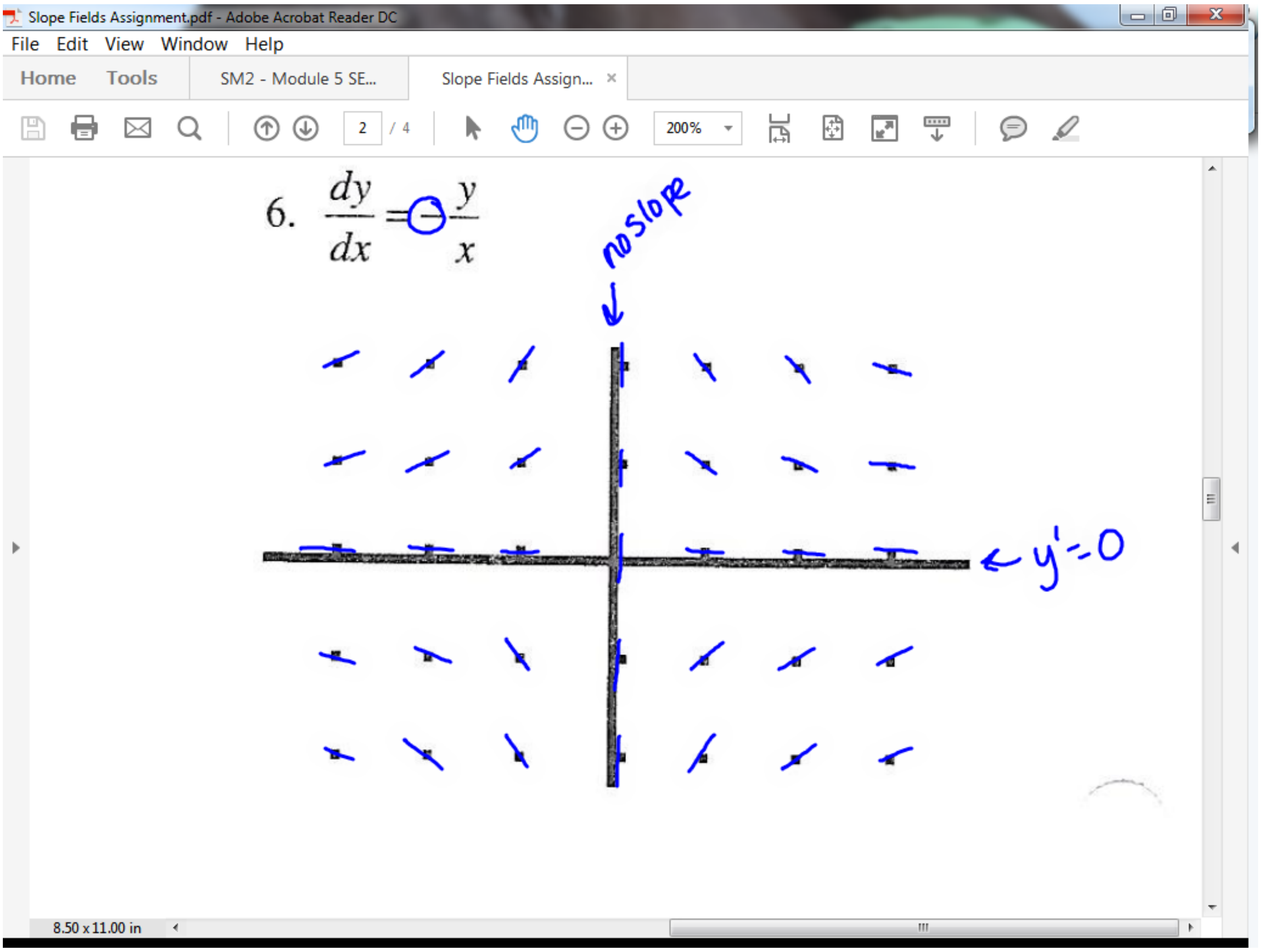
4. $\frac{dy}{dx} =$
8.50 x 11.00 in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6. dy/dx = -y/x
no slope
← y' = 0
8.50 x 11.00 in

Feb 9-9:36 AM

6. Figure 6 shows the slope field for the differential equation $\frac{dy}{dx} = \frac{-4x}{9y}$

From last time...

a) Calculate the slope at the point (2,-1). Mark this point on the graph. Does the calculated slope seem reasonable? Explain.

$$\frac{dy}{dx} = \frac{-4 \cdot 2}{9 \cdot -1} = \frac{-8}{-9} = \frac{8}{9}$$

b) Start at the point (0,2) and draw a graph that represents the particular solution of the differential equation that contains that point. Where does the curve seem to go after it touches the x-axis? What geometric shape does the figure seem to be?

ellipse

separation of variables

c) Solve the differential equation algebraically. Find the unique solution that contains the point (0,2). Does this graph agree with your guess in part b)?

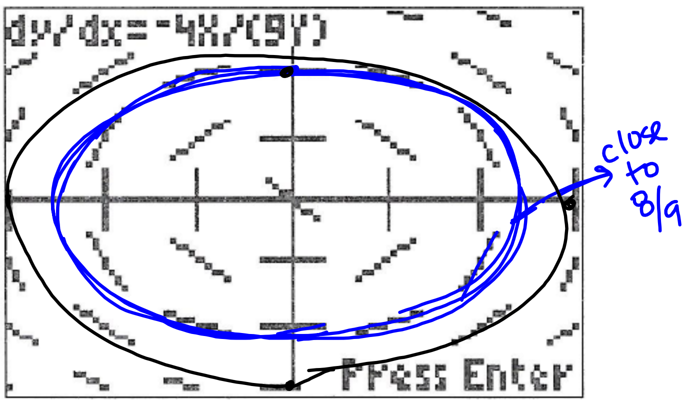


Figure 6

$$9y \cdot \frac{dy}{\cancel{dx}} = \frac{-4x}{9y} \cdot 9y \cdot dx$$

$$\int 9y\,dy = \int -4x\,dx$$

$$\frac{9y^2}{2} = -\frac{4x^2}{2} + C$$

$$\frac{9y^2}{2} = -2x^2 + C$$

$$(0,2) \rightarrow \frac{9(2)^2}{2} = -2(0)^2 + C$$

$$18 = C$$

$$\frac{9y^2}{2} = -2x^2 + 18$$

$$\frac{\frac{9y^2}{2}}{18} + \frac{2x^2}{18} = \frac{18}{18}$$

$$\frac{9y^2}{2 \cdot 18} + \frac{x^2}{9} = 1$$

$$\boxed{\frac{y^2}{4} + \frac{x^2}{9} = 1}$$

$$y^2 + \frac{4x^2}{9} = 4$$

$$\sqrt{y^2} = \sqrt{-\frac{4x^2}{9} + 4}$$

$$y = \pm\sqrt{\frac{-4x^2}{9} + 4}$$

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a: distance → or ← from y-axis

b: distance ↑ or ↓ from x-axis

On Calculator...

TI-NSpire

http://www.dummies.com/education/graphing-calculators/how-to-graph-differential-equations-on-ti-nspire/

TI-84

-I have to give you a program

Feb 6-2:18 PM

7.3 Slope Fields (7.1 in book)

Differential Equation

An equation involving a derivative is called a **differential equation**. The **order of a differential equation** is the order of the highest derivative involved in the equation.

First-order Differential Equation

If the general solution to a first-order differential equation is continuous, the only additional information needed to find a unique solution is the value of the function at a single point, called an **initial condition**. A differential equation with an initial condition is called an **initial-value problem**. It has a unique solution, called the **particular solution** to the differential equation.

Solve the following differential equations.

1. $\frac{dy}{dx} = \sin x$

$\int dy = \int \sin x \, dx$

$y = \cos x + C$

2. $\frac{dy}{dx} = \frac{2x}{y}$; the curve passes through the point (1, 2)

$\int y\,dy = \int 2x\,dx$

$\frac{y^2}{2} = \frac{2x^2}{2} + C$

$\frac{y^2}{2} = x^2 + C$

$(1,2) \rightarrow \frac{2^2}{2} = 1^2 + C$

$2 = 1 + C$

$1 = C$

$\frac{y^2}{2} = x^2 + 1$

$\sqrt{y^2} = \sqrt{2x^2+2}$

$\boxed{y = \sqrt{2x^2+2}}$

3. $\frac{dy}{dx} = 3y\cos x$; $y = 8$ when $x = 0$ (0,8)

$\cancel{dx} \cdot \frac{dy}{y \cdot \cancel{dx}} = 3\cos x \cdot dx$

$\int \frac{dy}{y} = \int 3\cos x\,dx$

$\ln|y| = 3\sin x + C$

$(0,8) \rightarrow \ln 8 = 3\sin 0 + C$

$\ln 8 = 0 + C$

$\ln 8 = C$

$\ln|y| = 3\sin x + \ln 8$
$\quad -\ln 8 \qquad\qquad -\ln 8$

$\ln|y| - \ln 8 = 3\sin x$

$\cancel{e}^{\cancel{\ln}\left(\frac{|y|}{8}\right)} = e^{3\sin x}$

$8 \cdot \frac{|y|}{8} = 8e^{3\sin x}$

$|y| = 8e^{3\sin x}$

check (0,8) $\boxed{y = 8e^{3\sin x}}$

$8 = 8\cancel{e^{3\sin 0}}$

$8 = 8$ ✓

$y = -8e^{3\sin x}$

$8 = -8\cancel{e^{3\sin 0}}$

$8 = -8$ X

NO!

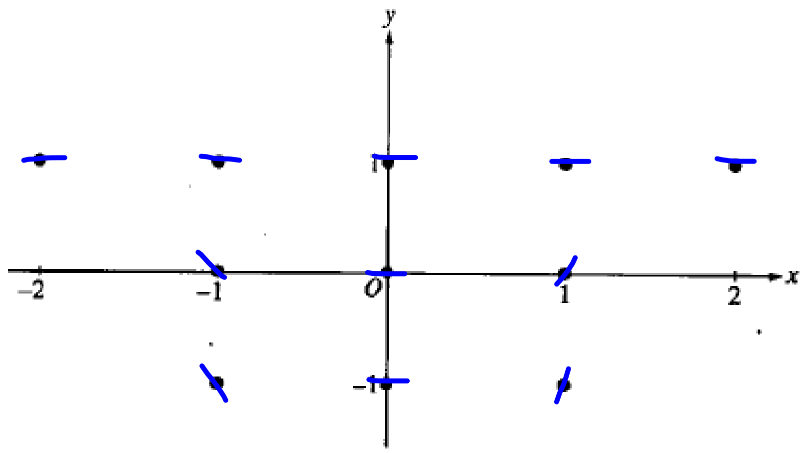
$y = 8e^{3\sin x}$

Feb 7-2:10 PM

4. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

$\frac{dy}{dx} = x(y-1)^2$

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

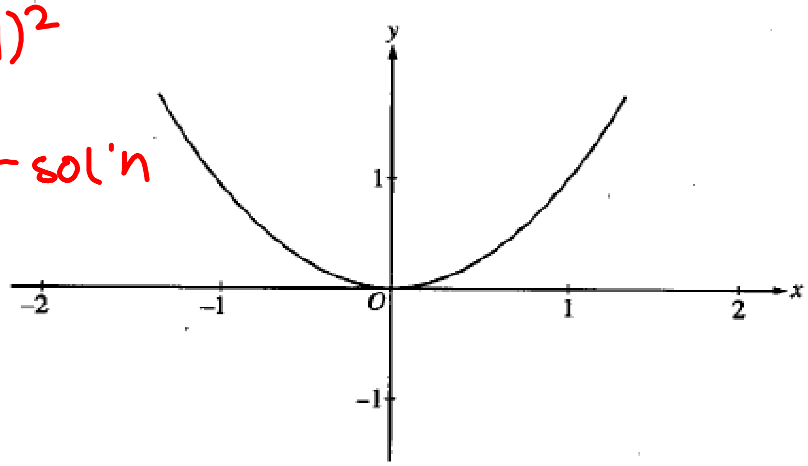


(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.

$\frac{dy}{dx} = x(y-1)^2$

particular sol'n

$f(0) = -1$



(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).

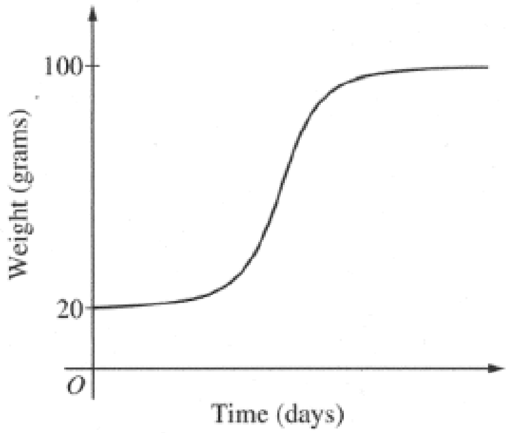
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Homework

7.6 Worksheet