7

Questions on 7.2 HW?

21 $\int_1^4 \frac{1}{1+x^3}\,dx$

$\frac{4-1}{3} = \frac{3}{3} = 1$ Δx or h

x	y
1	1/2
2	1/9
3	1/28
4	1/65

$T = \frac{1}{2}\cdot 1\left(\frac{1}{2} + 2\cdot\frac{1}{9} + 2\cdot\frac{1}{28} + \frac{1}{65}\right)$

$T = \frac{1}{2}\left(\frac{1}{2} + \frac{2}{9} + \frac{1}{14} + \frac{1}{65}\right)$

$T = \frac{1}{2}\left(\frac{3313}{4095}\right) \approx 0.4045 \approx \boxed{0.40}$

7 $\int (e^{2x}+2)^2\,dx = \int (e^{4x} + 4e^{2x} + 4)\,dx =$

$(e^{2x}+2)(e^{2x}+2)$

$e^{4x} + 4e^{2x} + 4$ ✓

$\int e^{4x}dx + \int 4e^{2x}dx + \int 4\,dx =$

$\boxed{\frac{1}{4}e^{4x} + 2e^{2x} + 4x + C}$

$u = 4x$
$du = 4dx$
$\frac{1}{4}du = dx$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$\int e^{4x} dx$

$\frac{1}{4}\int e^u du$

17 $a(t) = 3t^2 - 4t + 2$

$v(1) = 4$

$v(t) = \int (3t^2 - 4t + 2)\,dt$

$v(t) = \frac{3t^3}{3} - \frac{4t^2}{2} + 2t + C$

$v(t) = t^3 - 2t^2 + 2t + C$

$v(1) = 4$

$4 = (1)^3 - 2(1)^2 + 2(1) + C$

$4 = 1 - 2 + 2 + C$

$4 = 1 + C$

$3 = C$

$\boxed{v(t) = t^3 - 2t^2 + 2t + 3}$

7.3 Slope Fields (7.1 in book)

Differential Equation

An equation involving a derivative is called a **differential equation**. The **order of a differential equation** is the order of the highest derivative involved in the equation.

Example

Find all functions y that satisfy $\dfrac{dy}{dx} = 3x^2 + \cos x$.

$$y = \frac{3x^3}{3} + \sin x + C$$

$$y = x^3 + \sin x + C$$

The solution can be any antiderivative of $3x^2 + \cos x$, which can be any function of the form $y = x^3 + \sin x + C$.

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First-order Differential Equation

If the general solution to a first-order differential equation is continuous, the only additional information needed to find a unique solution is the value of the function at a single point, called an **initial condition**. A differential equation with an initial condition is called an **initial-value problem**. It has a unique solution, called the **particular solution** to the differential equation.

Example

$$y = \frac{1}{2}e^{2x} - \frac{3x^2}{2} + C$$ ← general sol'n

$$\left(1, \frac{1}{2}\right) \rightarrow \frac{1}{2} = \frac{1}{2}e^{2\cdot 1} - \frac{3(1)^2}{2} + C$$

$$\frac{1}{2} = \frac{1}{2}e^{2} - \cancel{\frac{3}{2}} + C$$

$$+\frac{3}{2} \qquad \cancel{+\frac{3}{2}}$$

$$2 = \frac{1}{2}e^{2} + C$$

$$\boxed{2 - \frac{1}{2}e^{2} = C}$$

Particular sol'n

$$\boxed{y = \frac{1}{2}e^{2x} - \frac{3}{2}x^2 + 2 - \frac{1}{2}e^2}$$

Find the particular solution to the equation $\frac{dy}{dx} = e^{2x} - 3x$ whose graph passes through the point $\left(1, \frac{1}{2}\right)$.

The general solution is $y = \frac{1}{2}e^{2x} - \frac{3}{2}x^2 + C$.

Applying the initial condition, we have $\frac{1}{2} = \frac{1}{2}e^2 - \frac{3}{2} + C$,

from which we conclude that

$C = 2 - \frac{1}{2}e^2$. Therefore, the particular equation is $y = \frac{1}{2}e^{2x} - \frac{3}{2}x^2 + 2 - \frac{1}{2}e^2$.

Slope Field

The differential equation gives the slope at any point (x, y). This information can be used to draw a small piece of the linearization at that point, which approximates the solution curve that passes through that point. Repeating that process at many points yields an approximation called a **slope field**.

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Example (if time)

Construct a slope field for the differential equation

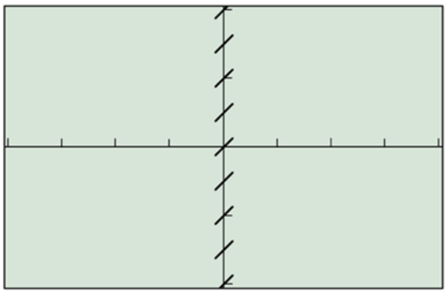
$$\frac{dy}{dx} = \cos x.$$

The slope at any point $(0, y)$ will be $\cos 0 = 1$.

The slope at any point (π, y) or $(-\pi, y)$ will be -1.

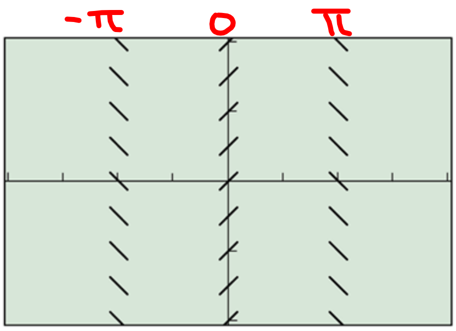
The slope at all odd multiples of $\frac{\pi}{2}$ will be 0.

The slope is 1 along the lines $x = \pm 2\pi$.



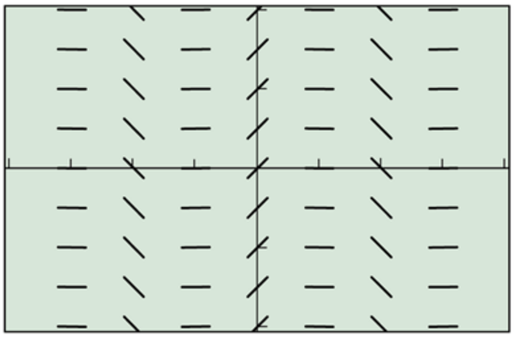
[−2π, 2π] by [−4, 4]

(a)



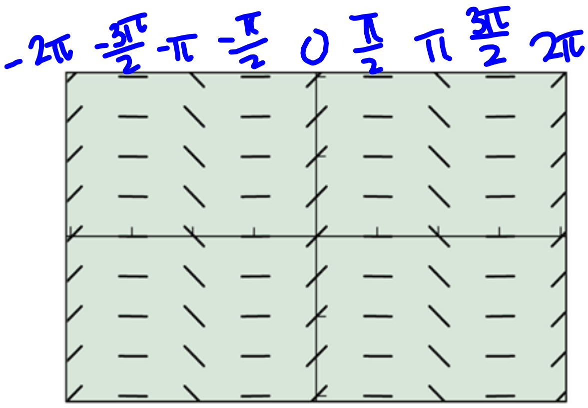
[−2π, 2π] by [−4, 4]

(b)



[−2π, 2π] by [−4, 4]

(c)



[−2π, 2π] by [−4, 4]

(d)

Slope Fields and Differential Equations

An equation like $\frac{dy}{dx} = y\ln x$, that contains a derivative is a differential equation. To determine **y** as a function of **x** when we are given its derivative and the value of the function at a given point is called an **initial value problem**. To **solve a differential equation** means to find the unique equation that satisfies the given conditions among the family of equations with the given derivative.

We can use graphical representations with slope fields and Euler's Method to identify the unique solution. The best way to understand slope fields is to draw some by hand. To do this, we draw small segments of tangents lines at selected points. We can do this because a differentiable function is locally linear at the point of tangency and can be approximated by its tangent line over a small interval.

1. Given the function: $y = 0.5x^2$. Write the derivative: $\frac{dy}{dx} = x$

At each grid point, calculate the value of the derivative and draw a short line segment with that slope.

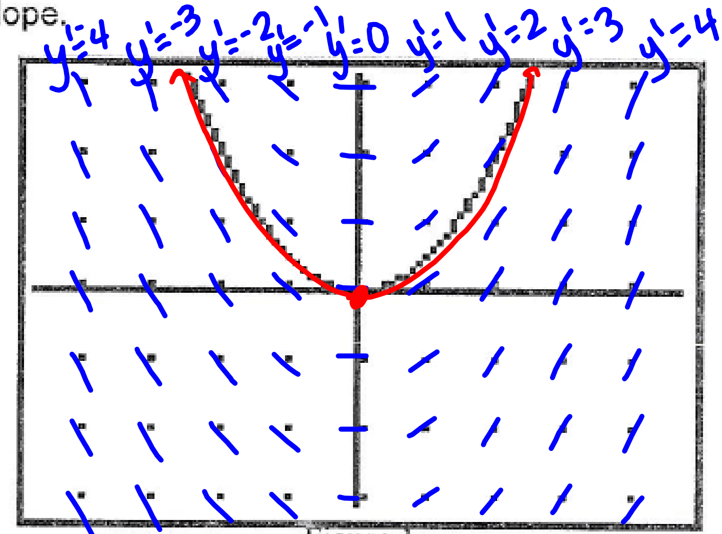


Figure 1

What family of functions seems to match all the slope fields?

$y = \frac{1}{2}x^2 + C$

(quadratic)

What is an initial condition of the function graphed?

(0,0)

$0 = \frac{1}{2}(0)^2 + C$

$0 = C$

$y = \frac{1}{2}x^2$

2. Sketch a function that matches this slope field.

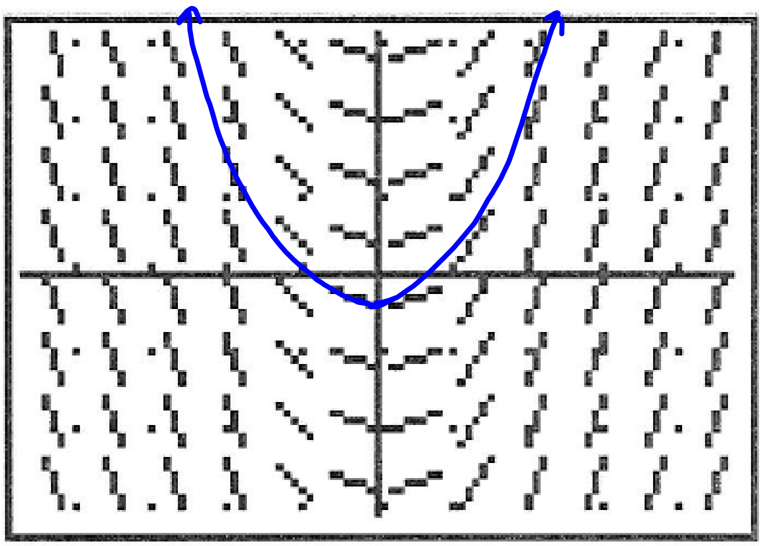
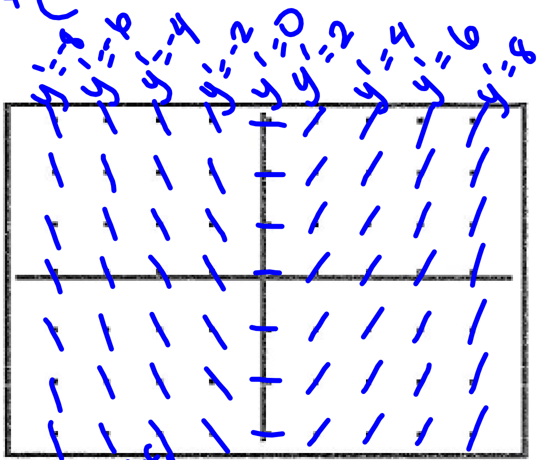


Figure 2

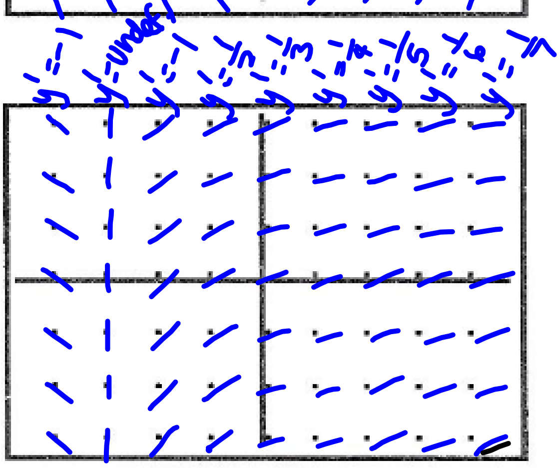
What family of functions seems to match this slope field?

$y = x^2 + C$

3. If $\frac{dy}{dx} = 2x$, sketch the slope field



4. If $\frac{dy}{dx} = \frac{1}{x+3}$, sketch the slope field.



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5.

a) Sketch the path of the unique solution if the graph passes through (0,1).

b) Sketch the path of the unique solution if the graph passes through (0,-1).

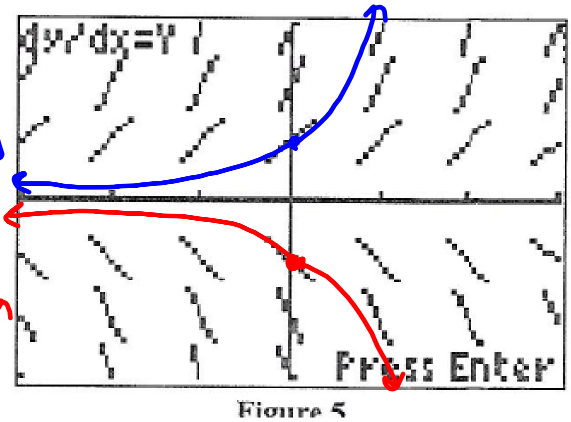

Figure 5

c) What familiar functions do these two graphs resemble?

exponential

d) Given $dx \cdot \frac{dy}{dx} = y \cdot dx$ verify your guess analytically

$$\frac{dy}{y} = \frac{y\,dx}{y}$$

$$\frac{dy}{y} = dx$$

$$\int \frac{dy}{y} = \int dx$$

$$\ln|y| = x + C$$

(0,1) $\ln 1 = 0 + C$

$$0 = C$$

$$\ln|y| = x + 0$$

$$e^{\ln|y|} = e^{x}$$

$$|y| = e^{x}$$

$\boxed{y = e^{x}}$ $y = -e^{x}$

$1 = e^{0}$ ~~$1 = -e^{0}$~~

$|x| = 2$
$x = 2$ and $x = -2$
$|3| = 3$
$|-3| = 3$

(1) Separate variables
y & dy on Ⓛeft
x, dx, coefficients on Ⓡight
• NEVER have dx or dy in denominator

(2) Integrate both sides

(3) +C on the x-side

(4) Solve for C using initial conditions

(5) Solve for y.

(6) If necessary, use initial conditions to choose the correct equation.

6. Figure 6 shows the slope field for the differential equation

$$\frac{dy}{dx} = \frac{-4x}{9y}$$

a) Calculate the slope at the point (2,-1). Mark this point on the graph. Does the calculated slope seem reasonable? Explain.

b) Start at the point (0,2) and draw a graph that represents the particular solution of the differential equation that contains that point. Where does the curve seem to go after it touches the x-axis? What geometric shape does the figure seem to be?

c) Solve the differential equation algebraically. Find the unique solution that contains the point (0,2). Does this graph agree with your guess in part b)?

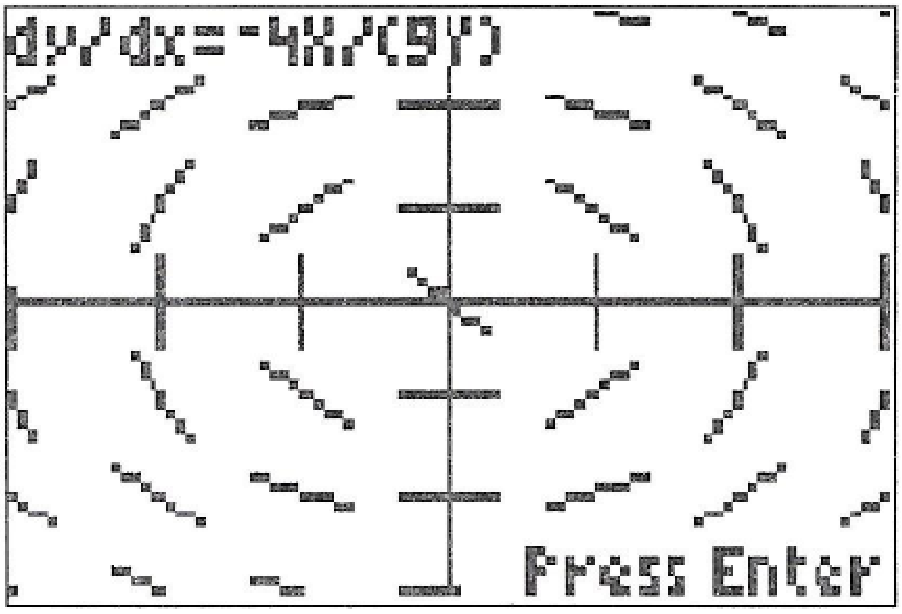

Figure 6

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http://www.dummies.com/education/graphing-calculators/how-to-graph-differential-equations-on-ti-nspire/

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Homework

Slope Fields Assignment Packet

Oct 21-8:47 AM