Get out your Unit 6 Outline and get ready to have your 6.7-6.11 HW checked off - it is due today!

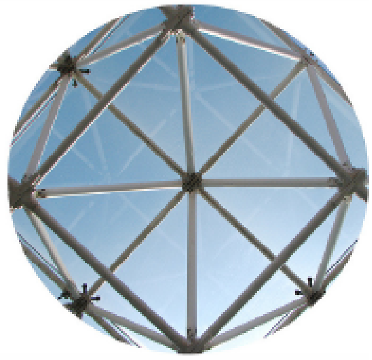
Also, I will be giving you points on what you finished from 7.1 in class Tuesday

Illuminate Block 3 Benchmark

-for participation points-

7.3 Cyclic Polygons

A Solidify Understanding Task



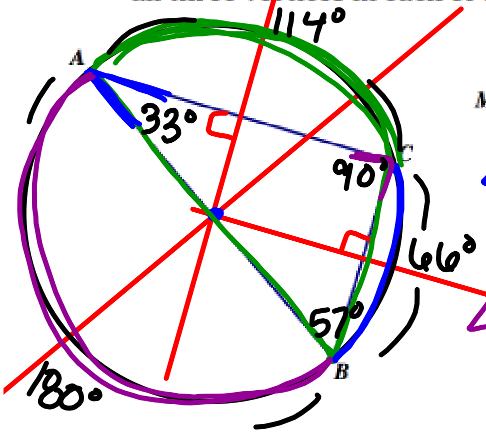
By definition, a cyclic polygon is a polygon that can be inscribed in a circle. That is, all of the vertices of the polygon lie on the same circle.

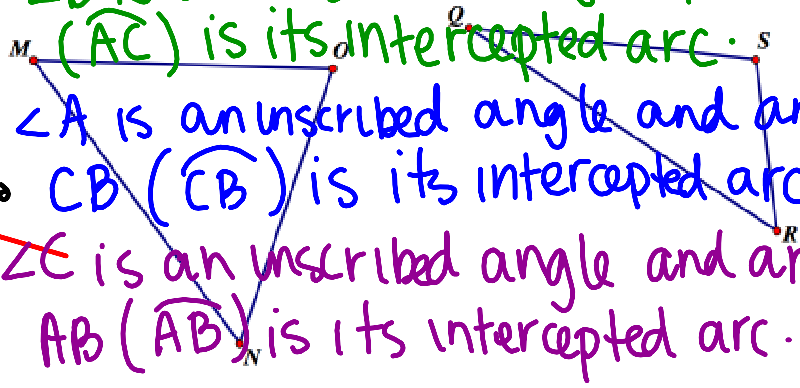
Part 1

In task *5.8 Centers of a Triangle* your work on Kara's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three vertices of any triangle, and therefore all triangles are cyclic polygons.

⊥ bisectors

1. Based on Kara's work, use a compass and straightedge to construct the circles that contain all three vertices in each of the following triangles.





$\angle B$ is an inscribed angle & arc AC ($\overset{\frown}{AC}$) is its intercepted arc.

$\angle A$ is an inscribed angle and arc CB ($\overset{\frown}{CB}$) is its intercepted arc.

$\angle C$ is an inscribed angle and arc AB ($\overset{\frown}{AB}$) is its intercepted arc.

Since each vertex of an inscribed triangle lies on the circle, each angle of the triangle is an inscribed angle. We know that the sum of the measures of the interior angles of the triangle is 180° and that the sum of the measures of the three intercepted arcs is 360°.

2. Using one of the diagrams of an inscribed triangle you created above, illustrate and explain why this last statement is true.

Because the three arcs cover the whole circle.

We know that the degree measure of an arc is, by definition, the same as the measure of the central angle formed by the radii that contain the endpoints of the arc. But how is the measure of an inscribed angle that intercepts this same arc related to the measure of the central angle and the intercepted arc? That is something useful to find out.

3. Using a protractor, find the measure of each arc represented on each circle diagram above. Then find the measure of each corresponding inscribed angle. Make a conjecture based on this data.

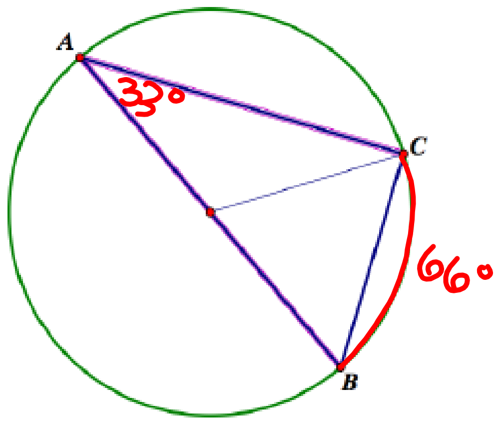
*My conjecture about the measure of an inscribed angle:
The measure of an inscribed angle is $\frac{1}{2}$ the measure of its intercepted arc.

The three circle diagrams you created above have been reproduced below. One inscribed angle has been bolded in each triangle. A diameter of the circle has also been added to each diagram as an auxiliary line segment, as well as some additional line segments that will assist in writing proofs about the inscribed angles. Three cases are illustrated: case 1, where the diameter is a side of the inscribed angle; case 2, where the diameter lies in the interior of the inscribed angle; and case 3, where the diameter lies in the exterior of the inscribed angle. In each diagram, prove your conjecture for the inscribed angle shown in bold.

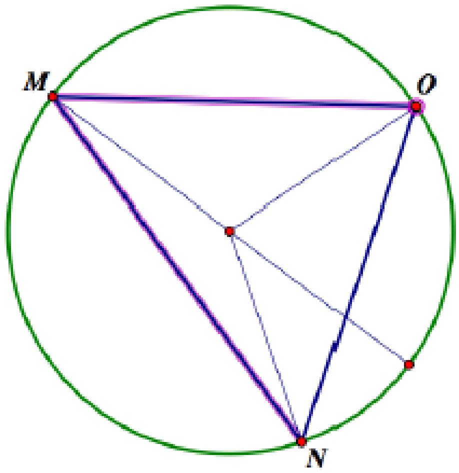
Case 1:

$m\overset{\frown}{CB} = 66°$

$m\angle A = \frac{1}{2} \cdot 66° = 33°$



Case 2:



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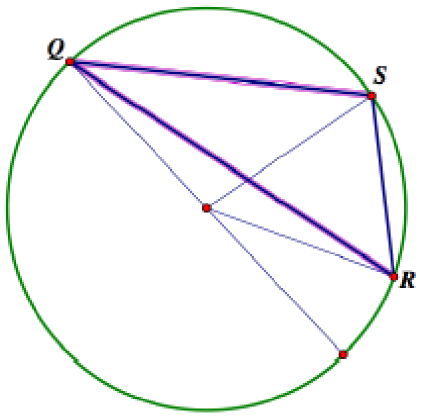
Case 3:



Part 2

We have found that all triangles are cyclic polygons. Now let's examine possible cyclic quadrilaterals.

4. Using dynamic geometry software, experiment with different types of quadrilaterals. Based on your experimentation, deicide which word best completes each of the following statements:

    a. [Some, all, no] squares are cyclic.

    b. [Some, all, no] rhombuses are cyclic.

    c. [Some, all, no] trapezoids are cyclic.

    d. [Some, all, no] rectangles are cyclic.

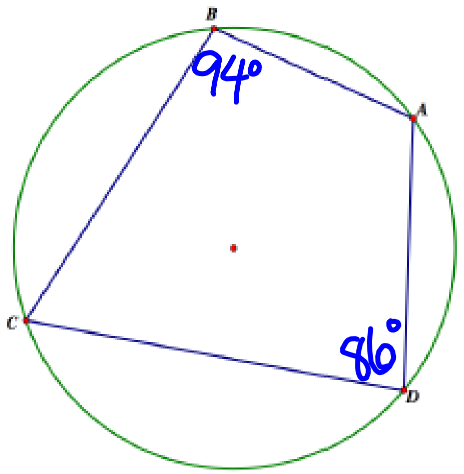
    e. [Some, all, no] parallelograms are cyclic.

Obviously, some generic quadrilaterals are cyclic, since you can select any four points on a circle as the vertices of a quadrilateral.

5. Using dynamic geometry software, experiment with cyclic quadrilaterals that are not parallelograms or trapezoids. Focus on the measurements of the angles. Make a conjecture about the measures of the angles of a cyclic quadrilateral. Then prove your conjecture using what you know about inscribed angles.

* My conjecture about the angles of a cyclic quadrilateral:
In a cyclic quadrilateral, opposite angles are supplementary (add to 180°).

Proof of my conjecture:
(How might you use the following diagram to assist you in your proof?)

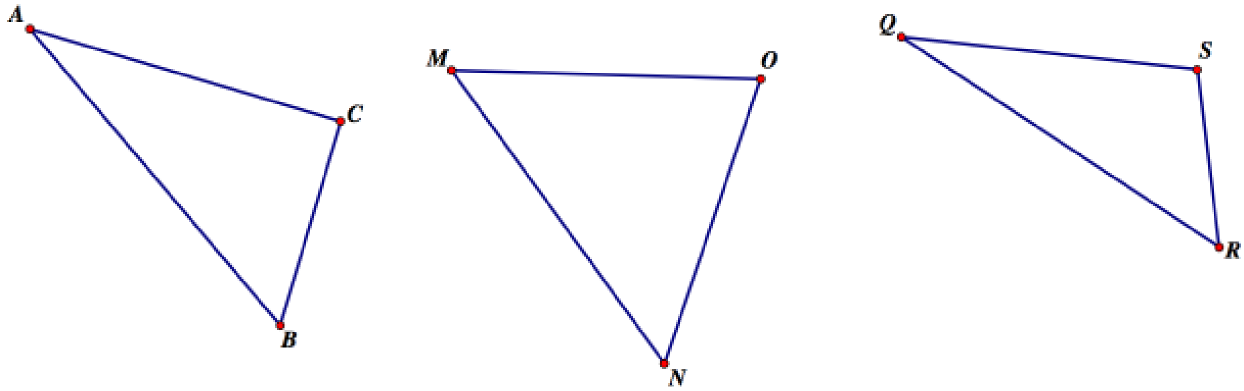


$\angle B$ & $\angle D$ are opposite $\angle$s
$m\angle B = 94°$ then
$m\angle D = 180° - 94° = 86°$

Part 3

In task *5.8 Centers of a Triangle*, your work on Kolton's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three sides of a triangle, and therefore a circle can be inscribed inside every triangle.

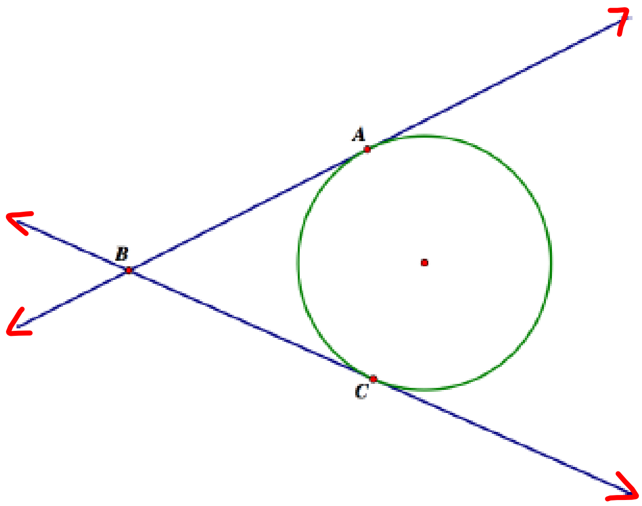
6. Based on Kolton's work, use a compass and straightedge to construct the circles that can be inscribed in each of the following triangles.



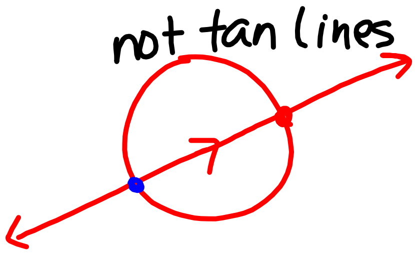
7. The angles of the triangle that are formed by the lines that are tangent to the circle are called circumscribed angles. Use dynamic geometry software to experiment with the measures of circumscribed angles relative to the arcs they intercept. Make a conjecture about the measures of the circumscribed angles. Then prove your conjecture using what you know about inscribed angles.

My conjecture about the measures of circumscribed angles:

Proof of my conjecture:



Tangent lines to a circle are lines that intersect the circle exactly once.



8. Based on your work in this task and the previous task, describe a procedure for constructing a tangent line to a circle through a given point outside the circle.

Homework

Finish 7.3 "Ready, Set, Go"