Questions on 7.2 HW?

Nov 3-2:38 PM

7.2b More Integration by Substitution

Tricky inverse trig u-subs:

$$\int \frac{dx}{4+x^2} = \int \frac{dx}{4\left(1+\frac{x^2}{4}\right)} = \frac{1}{4}\int \frac{dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2}\int \frac{du}{1+u^2} =$$

$\frac{1}{2}$ $\frac{1}{2}$

$$\frac{1}{2}\arctan u + C = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$du = \frac{1}{2}dx$

$$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{9\left(1-\frac{x^2}{9}\right)}} = \int \frac{dx}{3\sqrt{1-\left(\frac{x}{3}\right)^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}} =$$

du

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$$= \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin\left(\frac{x}{3}\right)+C$$

$$\int \frac{dx}{x^2+16} = \int \frac{dx}{16\left(\frac{x^2}{16}+1\right)} = \frac{1}{16}\int \frac{dx}{\left(\frac{x}{4}\right)^2+1} = \frac{1}{4}\int \frac{du}{u^2+1}$$

$\frac{1}{4}\cdot\frac{1}{4}$

$u = \frac{x}{4}$

$du = \frac{1}{4}dx$

$$= \frac{1}{4}\arctan u + C = \frac{1}{4}\arctan\left(\frac{x}{4}\right)+C$$

U-Substitution with Definite Integrals

Evaluate $\int_0^2 \frac{x}{x^2-9}\,dx$.

$u = x^2 - 9$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u(2) = 2^2 - 9 = -5$

$u(0) = 0^2 - 9 = -9$

$= \frac{1}{2}\int_{-9}^{-5} \frac{du}{u} = \frac{1}{2}\left[\ln|u|\right]_{-9}^{-5}$

$= \frac{1}{2}\left[\ln|-5| - \ln|-9|\right]$

$= \frac{1}{2}(\ln 5 - \ln 9) =$

$\frac{1}{2}\left(\ln\frac{5}{9}\right) = -0.293$

Let $u = x^2 - 9$ and $du = 2x\,dx$. Then $u(0) = 0^2 - 9 = -9$ and $u(2) = 2^2 - 9 = -5$. So,

$$\int_0^2 \frac{x}{x^2-9}\,dx = \frac{1}{2}\int_{-9}^{-5} \frac{du}{u}$$

$$= \frac{1}{2}\ln|u|\Big|_{-9}^{-5}$$

$$= \frac{1}{2}(\ln 5 - \ln 9)$$

$$= \frac{1}{2}\ln\left(\frac{5}{9}\right)$$

Examples

$\int_0^{\frac{3\pi}{2}} \sin^2 x \cos x \, dx =$ (du)

$u = \sin x$

$du = \cos x \, dx$

$u(\frac{3\pi}{2}) = \sin(\frac{3\pi}{2}) = -1$

$u(0) = \sin(0) = 0$

$= \int_0^{-1} u^2 du = -\int_{-1}^{0} u^2 du$

$= -\left[\frac{u^3}{3}\right]_{-1}^{0}$

$= -\left(0 - \frac{-1}{3}\right) = -\left(\frac{1}{3}\right)$

$= -\frac{1}{3}$

$\int_2^3 \frac{x}{x^2+1} dx =$

$u = x^2 + 1$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$u(3) = 3^2 + 1 = 10$

$u(2) = 2^2 + 1 = 5$

$= \frac{1}{2}\int_5^{10} \frac{1}{u} du = \frac{1}{2}\left[\ln|u|\right]_5^{10}$

$= \frac{1}{2}(\ln 10 - \ln 5) =$

$= \frac{1}{2}\ln\left(\frac{10}{5}\right) = \frac{1}{2}\ln 2$

≈ 0.3466

$\int_{-1}^{2} x^2(x^3+4) dx =$

$u = x^3 + 4$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$u(2) = 2^3 + 4 = 12$

$u(-1) = (-1)^3 + 4 = 3$

$= \frac{1}{3}\int_3^{12} u \, du = \frac{1}{3}\left[\frac{u^2}{2}\right]_3^{12} = \frac{1}{3}\left(\frac{144}{2} - \frac{9}{2}\right)$

$= \frac{1}{3}\left(\frac{135}{2}\right) = \frac{135}{6}$

$= 22.5$

Homework

7.2b Worksheet

Oct 21-8:47 AM