Questions on 7.1 WKS? Remember, we are temporarily skipping 11,13,14,15...

17 $\int 5x^{-1}\,dx = 5\int \frac{1}{x}\,dx = \boxed{5\ln|x|+C}$

7.2 Integration by Substitution

Remember these...?

15 Integrals you MUST memorize!

$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$, when $n \neq -1$

$\int a^u \ln a \, du = \frac{a^u}{\ln a} + C$

$\int e^u \, du = e^u + C$

$\int \sin u \, du = -\cos u + C$

$\int \cos u \, du = \sin u + C$

$\int \sec^2 u \, du = \tan u + C$

$\int \csc^2 u \, du = -\cot u + C$

$\int \sec u \tan u \, du = \sec u + C$

$\int \csc u \cot u \, du = -\csc u + C$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \frac{du}{u} = \ln|u| + C$

$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$

$\int \frac{du}{1+u^2} = \arctan u + C$

$\int \frac{du}{u \ln a} = \log_a u + C$

Example Using Substitution

Evaluate $\int x^2 e^{x^3} dx.$ =

$u = x^3$

$\cancel{dx} \cdot \frac{du}{\cancel{dx}} = 3x^2 \cdot dx$

$\frac{du}{3} = \frac{3x^2 dx}{3}$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3}\int e^u du = \frac{1}{3} e^u + C$

$= \boxed{\frac{1}{3} e^{x^3} + C}$

Let $u = x^3$. Then $\frac{du}{dx} = 3x^2$, from which we conclude that

$\frac{1}{3} du = x^2 dx$. We rewrite the integral and proceed as follows

$$\int x^2 e^{x^3} dx = \frac{1}{3}\int e^u du$$
$$= \frac{1}{3} e^u + C$$
$$= \frac{1}{3} e^{x^3} + C$$

Example Using Substitution

Evaluate $\int 6x\sqrt{1+x^2}\,dx$.

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

need 6x ... mult. int. by 3

$3du = 6x\,dx$

$= 3\int \sqrt{u}\,du =$

$3\int u^{1/2}du = \frac{3u^{3/2}}{3/2} + C$

$3 \cdot \frac{2}{3} \cdot u^{3/2} + C = 2u^{3/2} + C = \boxed{2(1+x^2)^{3/2} + C}$

Let $u = 1+x^2$. Then $du = 2xdx$. Rewrite the integral in terms of u :

$$\int 6x\sqrt{1+x^2}\,dx = 3\int \sqrt{u}\,du$$

$$= 3\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$$

$$= 2\left(1+x^2\right)^{\frac{3}{2}} + C$$

Example Using Substitution

$\cos^2x+\sin^2x=1$
$1-\cos^2x=\sin^2x$
$1-\sin^2x=\cos^2x$

Evaluate $\int \sin^3 x dx$. $=\int(\sin^2x)(\sin x)dx$

$u=\cos x$

$\frac{du}{dx}=-\sin x$

$du=-\sin x dx$

$-du=\sin x dx$

$=\int(1-\cos^2x)\sin x\,dx$

$=-\int(1-u^2)du=-\left(u-\frac{u^3}{3}+C\right)$

$=-u+\frac{1}{3}u^3+C=\boxed{-\cos x+\frac{1}{3}\cos^3x+C}$

$$\int \sin^3 x dx=\int(\sin^2 x)\sin x dx$$
$$=\int(1-\cos^2 x)\sin x dx$$
$$=-\int(1-u^2)du \qquad \text{let } u=\cos x \text{ and } -du=\sin x dx$$
$$=-u+\frac{u^3}{3}+C$$
$$=-\cos x+\frac{\cos^3 x}{3}+C$$

Example Using Substitution

Evaluate $\int_0^2 \frac{x}{x^2-9}\,dx.$

Let $u = x^2 - 9$ and $du = 2x\,dx$. Then $u(0) = 0^2 - 9 = -9$ and $u(2) = 2^2 - 9 = -5$. So,

$$\int_0^2 \frac{x}{x^2-9}\,dx = \frac{1}{2}\int_{-9}^{-5} \frac{du}{u}$$

$$= \frac{1}{2}\ln|u|\Big|_{-9}^{-5}$$

$$= \frac{1}{2}(\ln 5 - \ln 9)$$

$$= \frac{1}{2}\ln\left(\frac{5}{9}\right)$$

You try...

$$\int (x^2+2x-3)^2 (x+1)\,dx = \frac{1}{2}\int u^2\,du =$$

$$\frac{1}{2}\cdot\frac{u^3}{3} + C = \frac{1}{6}u^3 + C =$$

$$\boxed{\frac{1}{6}(x^2+2x-3)^3 + C}$$

$u = x^2+2x-3$

$\frac{du}{dx} = 2x+2$

$du = 2x+2\,dx$

$\frac{du}{2} = \frac{2(x+1)dx}{2}$

$\frac{1}{2}du = (x+1)dx$

$$\int 2\tan^2 x \sec^2 x\,dx =$$

$$2\int u^2\,du = 2\cdot\frac{u^3}{3} + C$$

$$= \frac{2}{3}u^3 + C = \boxed{\frac{2}{3}\tan^3 x + C}$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$$\int \sqrt{\sin x}\cos x\,dx =$$

$$\int \sqrt{u}\,du = \int u^{1/2}\,du$$

$$= \frac{2}{3}\cdot u^{3/2} + C = \boxed{\frac{2}{3}(\sin x)^{3/2} + C \text{ or } \frac{2}{3}\sqrt{\sin^3 x} + C}$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

How would you integrate the following pairs of integrals? What is different about the methods you use for each of them?

$\int \frac{dx}{1+x}$ and $\int \frac{dx}{(1+x)^2}$

$u = 1+x$

$\frac{du}{dx} = 1$

$du = dx$

$\rightarrow \int \frac{1}{u} du = \int \frac{du}{u} = \ln|u| + C$

more a rule

$= \boxed{\ln|x+1| + C}$

$\rightarrow$ $u = 1+x$, $\frac{du}{dx} = 1$, $du = dx$

$\rightarrow \int \frac{1}{u^2} du = \int u^{-2} du$

$= \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C$

$= \boxed{-\frac{1}{1+x} + C}$

substitution

$\int \frac{4}{x^2+1} dx$ and $\int \frac{x}{x^2+1} dx$

$4\int \frac{1}{x^2+1} dx = 4\arctan x + C$

RULE! :)

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{1}{2} du = x\,dx$

$\rightarrow \frac{1}{2}\int \frac{1}{u} du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

$= \boxed{\frac{1}{2}\ln(x^2+1) + C}$

SUB! POWER RULE

$\int 2x(x^2+1)^2 dx$ and $\int (x^2+1)^2 dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\rightarrow \int u^2 du$

$= \frac{u^3}{3} + C$

$= \boxed{\frac{(x^2+1)^3}{3} + C}$

SUB $\rightarrow$

$\int (x^2+1)(x^2+1) dx =$

$\int (x^4 + 2x^2 + 1) dx =$

$\boxed{\frac{x^5}{5} + \frac{2x^3}{3} + x + C}$

Jan 31-8:53 AM

Homework

7.2: pgs.342-343 #1-11odds, 15-45 (X3)