If you haven't checked off your unit 6 homework, get that ready to be checked off! We will start unit 7 today! :)

Also, turn in your Unit 6 Review if you haven't already

7.1 Indefinite Integrals

REMEMBER: A function $F(x)$ is an antiderivative of a function $f(x)$ if $F'(x) = f(x)$.

What is the indefinite integral of a function f?

antiderivative; no upper or lower limit of integration.

$$y = \int f(x)\,dx = F(x) + C$$

What is the difference between a definite integral and an indefinite integral?

definite integrals have an upper & lower limit of integration; indefinite integrals do not

If $F(x)$ is an antiderivative of $f(x)$, then $\int f(x)dx =$ $\underline{F(x) + C}$.

Why do you have to include '+ C'?

The derivative of a constant, C, is zero.

What do you need in order to find the specific value of C when you integrate?

An (x,y) point from F(x).

EXAMPLE: Find $g(x)$ if $g(x)=\int 2x\,dx$ and $g(1)=-6$.

$$y=\int 2x\,dx = \frac{2x^2}{2}+C = x^2+C$$

$$y=x^2+C$$

$$\rightarrow g(1)=-6$$

$$-6=(1)^2+C$$

$$-6=1+C$$

$$-1 \quad -1$$

$$-7=C$$

$$y=x^2-7$$

15 Integrals you MUST memorize!

$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$, when $n \neq -1$

$\int a^u \ln a \, du = \frac{a^u}{\ln a} + C$

$\int e^u \, du = e^u + C$

$\int \sin u \, du = -\cos u + C$

$\int \cos u \, du = \sin u + C$

$\int \sec^2 u \, du = \tan u + C$

$\int \csc^2 u \, du = -\cot u + C$

$\int \sec u \tan u \, du = \sec u + C$

$\int \csc u \cot u \, du = -\csc u + C$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \frac{du}{u} = \ln|u| + C$

$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$

$\int \frac{du}{1+u^2} = \arctan u + C$

$\int \frac{du}{u \ln a} = \log_a u + C$

Examples: Find the following:

1. $\int x^5\,dx = \frac{x^6}{6} + C$

2. $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = \boxed{2x^{1/2} + C \text{ or } 2\sqrt{x} + C}$

3. $\int e^{-3x}\,dx =$

substitution

$u = -3x$

$\frac{du}{-3} = \frac{-3\,dx}{-3}$

$\frac{du}{-3} = dx$

$\boxed{-\frac{1}{3}du = dx}$

$-\frac{1}{3}\int e^u\,du =$

$-\frac{1}{3}(e^u + C) =$

$-\frac{1}{3}(e^{-3x} + C)$

$-\frac{1}{3}e^{-3x} + C$

4. $\int \cos\frac{x}{2}\,dx =$

$u = \frac{x}{2} = \frac{1}{2}x$

$u = \frac{1}{2}x$

$\frac{du}{\frac{1}{2}} = \frac{\frac{1}{2}dx}{\frac{1}{2}}$

$\boxed{2du = dx}$

$2\int \cos u\,du =$

$2(\sin u + C) =$

$\boxed{2\sin\left(\frac{x}{2}\right) + C}$

Properties of indefinite integrals:

→ $\int k\cdot f(x)\,dx = k\int f(x)\,dx$, for any constant k.

→ $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Examples (cont'd):

Pay attention to the differential (*dx* or *du*)!

$f(u) = u^3 + 1$

Given $f(x) = x^3 + 1$ and $\underline{u = x^2}$, find the following:

$\int f(x)\,dx = \int (x^3 + 1)\,dx = \boxed{\frac{x^4}{4} + x + C}$

$\int f(u)\,du = \int (u^3 + 1)\,du = \frac{u^4}{4} + u + C$

$= \frac{(x^2)^4}{4} + x^2 + C$

$= \boxed{\frac{x^8}{4} + x^2 + C}$

$\int f(u)\,dx = \int (u^3 + 1)\,dx =$

$\int ((x^2)^3 + 1)\,dx = \int (x^6 + 1)\,dx =$

$\boxed{\frac{x^7}{7} + x + C}$

Homework

7.1Worksheet: Indefinite Integrals