Questions on Lesson 6.5?

Find all the possible rational roots of the following polynomial using the rational root theorem and factor this completely and solve

3. Use the Rational Root Theorem to solve $2x^4 - 9x^3 + 11x^2 - 9x + 9 = 0$.

q p

$p: \pm\{1, 3, 9\}$ $q: \pm\{1, 2\}$

$\frac{p}{q} \to \pm\{\frac{1}{1}, \frac{3}{1}, \frac{9}{1}, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}\}$

SIMPLIFY

not a factor $(x-1)$ $\pm\{1, 3, 9, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}\}$

not a root

1	2	-9	11	-9	9
	↓	2	-7	4	-5
	2	-7	4	-5	4

$(x-3)$ factor root

3	2	-9	11	-9	9
	↓	6	-9	6	-9
	2	-3	2	-3	$\boxed{0}$

$2x^4 - 9x^3 + 11x^2 - 9x + 9$

$= (x-3)(2x^3 - 3x^2 + 2x - 3)$

$q: \pm\{1, 2\}$ $p: \pm\{1, 3\}$

$\frac{p}{q}: \pm\{\frac{1}{1}, \frac{3}{1}, \frac{1}{2}, \frac{3}{2}\} = \pm\{1, 3, \frac{1}{2}, \frac{3}{2}\}$

$(2x-3)$

$\frac{3}{2}$	2	-3	2	-3
	↓	3	0	3
	2	0	2	$\boxed{0}$

$2(x - \frac{3}{2})2$

$(2x-3)$

$2x^4 - 9x^3 + 11x^2 - 9x + 9$

$= (x-3)(2x-3)(2x^2+2) = 0$

$= \boxed{(x-3)(2x-3)(x+i)(x-i)}$ fully factored

$2x^2 + 2 = 0$
$-2 \quad -2$

$\frac{2x^2}{2} = \frac{-2}{2}$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$\boxed{x = 3, \frac{3}{2}, -i, i}$ solutions

6.7 The Curious Case of Pascal's Triangle

Pascal's Triangle and the Binomial Theorem

pg.493-494 in your book.

There is an interesting pattern of numbers that makes up what is referred to as Pascal's Triangle.

The first six rows of Pascal's Triangle are shown, where $n = 0$ represents the first row, $n = 1$ represents the second row, and so on.

$n = 0 \rightarrow$ 1

$n = 1 \rightarrow$ 1 1

$n = 2 \rightarrow$ 1 2 1

$n = 3 \rightarrow$ 1 3 3 1

$n = 4 \rightarrow$ 1 4 6 4 1

$n = 5 \rightarrow$ 1 5 10 10 5 1

$n = 6 \rightarrow$ 1 6 15 20 15 6 1

$n = 7 \rightarrow$ 1 7 21 35 35 21 7 1

1. Analyze the patterns in Pascal's Triangle.

   a. Describe all the patterns you see in Pascal's Triangle.

   b. Complete the rows for $n = 6$ and $n = 7$ in the diagram of Pascal's Triangle. Describe the pattern you used.

pg.497 in your book

The patterns shown in Pascal's Triangle have many uses. For instance, you may have used Pascal's Triangle to calculate probabilities. Let's explore how you can use Pascal's Triangle to raise a binomial to a positive integer.

5. Multiply to expand each binomial. Write your final answer so that the powers of *a* are in descending order.

   a. $(a + b)^0 =$ 1

   b. $(a + b)^1 =$ $1a + 1b$

   c. $(a + b)^2 =$ $1a^2 + 2ab + 1b^2$

   d. $(a + b)^3 =$ $(a+b)(a^2+2ab+b^2) =$
   $1a^3 + 3a^2b + 3ab^2 + 1b^3$

   e. $(a + b)^4 =$ $(a+b)(a^3+3a^2b+3ab^2+b^3) =$
   $1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4a^0$

* a's exponents decrease by 1 as we go L to R.
  → begin with n from $(a+b)^n$, end with 0.
* b's exponents increase by 1 as we go L to R.
  → begin with 0 & end with the n from $(a+b)^n$

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pg.498 in your book

6. Analyze your answers to Question 5.

   a. Compare the coefficients of each product with the numbers shown in Pascal's Triangle. What do you notice?

   b. What do you notice about the exponents of the *a*- and *b*-variables in each expansion?

   c. What do you notice about the sum of the exponents of the *a*- and *b*-variables in each expansion?

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7. Use Pascal's Triangle to expand each binomial.

   row 5 in Pascal's △
   1 5 10 10 5 1

   a. $(a + b)^5 =$

   $1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5$

   row6:
   1 6 15 20 15 6 1

   b. $(a + b)^6 =$

   $1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6a^1b^5 + 1a^0b^6$

   c. $(a + b)^7 =$

   7th row: 1 7 21 35 35 21 7 1

   $1a^7b^0 + 7a^6b^1 + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7a^1b^6 + 1a^0b^7$

pg.499 in your book

What if you want to expand a binomial such as $(a + b)^{15}$? You could take the time to draw that many rows of Pascal's Triangle, but there is a more efficient way.

Recall that the factorial of a whole number n, represented as $n!$, is the product of all numbers from 1 to n.

1. Perform each calculation and simplify.

   a. $5! =$

   c. $\frac{5!}{3!} =$

You may have seen the notation $\binom{n}{k}$ or ${}_nC_k$ when calculating probabilities in another course. Both notations represent the formula for a *combination*. Recall that a combination is a selection of objects from a collection in which order does not matter. The formula for a combination of k objects from a set of n objects for $n \geq k$ is shown.

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$$

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Calculate $\binom{4}{2}$, or ${}_4C_2$.

$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$	Write the formula for a combination.
$n = 4$ and $k = 2$	Identify n and k.
$\binom{4}{2} = \frac{4!}{2!(4-2)!}$	Substitute the values for n and k into the formula.
$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)\,(2 \cdot 1)}$	Write each factorial as a product.
$= \frac{4 \cdot 3 \cdot \cancel{2 \cdot 1}}{(2 \cdot 1)\,(\cancel{2 \cdot 1})}$	Divide out common factors.
$= \frac{12}{2} = 6$	Simplify.

3. Perform each calculation and simplify.

a. $\binom{5}{1} =$

b. ${}_7C_4 =$

pg.501 in your book

The **Binomial Theorem** states that it is possible to extend any power of $(a + b)$ into a sum of the form shown.

$$(a + b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n$$

5. Use the Binomial Theorem to expand $(a + b)^{15}$. You can use your calculator to determine the coefficients.

$(a + b)^{15} =$

pg.502 in your book

Suppose you have a binomial with coefficients other than one, such as $(2x + 3y)^5$. You can use substitution along with the Binomial Theorem to expand the binomial.

You can use the Binomial Theorem to expand $(a + b)^5$, as shown.

$$(a + b)^5 = \binom{5}{0}a^5b^0 + \binom{5}{1}a^4b^1 + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}a^1b^4 + \binom{5}{5}a^0b^5$$

$$= a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + b^5$$

Now consider $(2x + 3y)^5$.

Let $2x = a$ and let $3y = b$.

You can substitute $2x$ for a and $3y$ for b into the expansion for $(a + b)^5$.

$$(2x + 3y)^5 = (2x)^5 + 5(2x)^4(3y)^1 + 10(2x)^3(3y)^2 + 10(2x)^2(3y)^3 + 5(2x)^1(3y)^4 + (3y)^5$$

$$= 32x^5 + 5(16x^4)(3y) + 10(8x^3)(9y^2) + 10(4x^2)(27y^3) + 5(2x)(81y^4) + 243y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

pg.502 in your book

6. Use the Binomial Theorem and substitution to expand each binomial.

a. $(3x + y)^4$

$(a+b)^n$

$a = 3x$

$b = y$

$n = 4$

Find 3rd term:

4th row: 1 4 6 4 1

coefficient

3rd term:

$6a^2b^2$

$6(3x)^2(y)^2 = 6 \cdot 9x^2 \cdot y^2 = 54x^2y^2$

$1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4ab^3 + 1b^4a^0$

$1(3x)^4(y)^0 + 4(3x)^3(y)^1 + 6(3x)^2(y)^2 + 4(3x)^1(y)^3 + 1(3x)^0(y)^4 =$

$$\boxed{81x^4 + 108x^3y + 54x^2y^2 + 12xy^3 + y^4}$$

Dec 10-8:30 AM

Lesson 6.7 Problems
-not in your book-

1. Consider $(v + w)^8$.
   a. Use Pascal's Triangle to expand $(v + w)^8$.
   b. Determine the coefficient of v^5w^3 in the expansion of $(v + w)^8$.
   c. Determine the coefficient of v^5w^3 in the expansion of $(2v + w)^8$.
   d. Determine the coefficient of v^4w^4 in the expansion of $(2v + 3w)^8$.
2. Expand $(4x + 2y)^5$.
3. Expand $(3m - n)^6$.
4. Expand $(-5x - 3y)^4$.
5. Determine the coefficient of c^5d^4 in the expansion of $(2c + 3d)^9$.
6. Determine the coefficient of j^7k^3 in the expansion of $(2j - k)^{10}$.

Homework
Finish Lesson 6.5