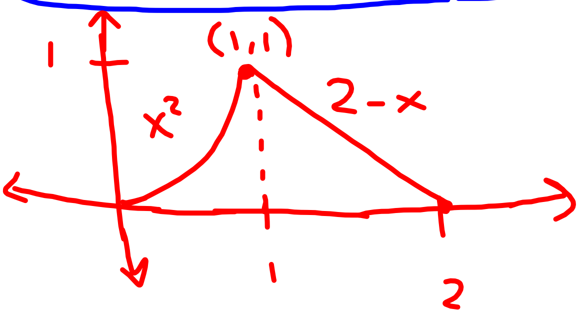
Questions on 6.4 HW? We'll finish up test corrections after the 6.5 lesson.

18 $y = \int_{3x^2}^{5x} \ln(2+p^2)\,dp = \frac{d}{dx}\int_0^{5x} \ln(2+p^2)\,dp - \frac{d}{dx}\int_0^{3x^2} \ln(2+p^2)\,dp =$

$$\boxed{5\ln(2+25x^2) - 6x\ln(2+9x^4)}$$

45



$\int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx =$

$\left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 =$

$\left[\frac{1}{3} - 0\right] + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right] =$

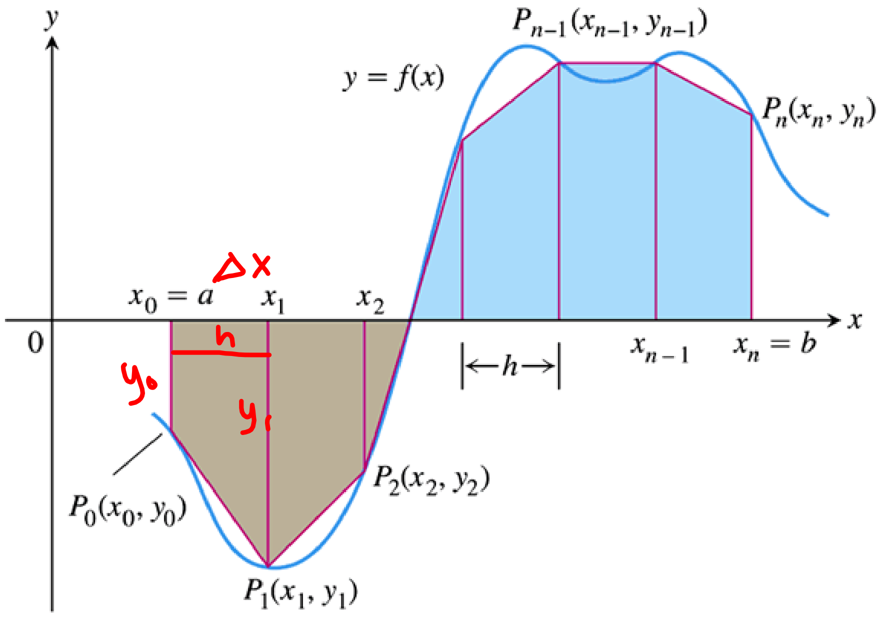
$\frac{1}{3} + 2 - 1\frac{1}{2} = 2\frac{1}{3} - 1\frac{1}{2} = 5/6$

6.5 Trapezoidal Rule

MRAM is more efficient in approximating integrals than either LRAM or RRAM; however, all three RAM approximations depend of the areas of the rectangles.

Are there other geometric shapes with known areas that can do the job more efficiently? YES! Trapezoids!

If [a,b] is partitioned into n subintervals of equal length h=(b-a)/n, the graph of f on [a,b] can be approximated by a straight line segment over each subinterval.



$$\int_a^b f(x)dx \approx h \cdot \frac{y_0 + y_1}{2} + h \cdot \frac{y_1 + y_2}{2} + \ldots + h \cdot \frac{y_{n-1} + y_n}{2}$$

$$= h\left(\frac{y_0}{2} + y_1 + y_2 + \ldots + y_{n-1} + \frac{y_n}{2}\right)$$

$$= \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n),$$

where $y_0 = f(a), \quad y_1 = f(x_1), \ \ldots, \ y_{n-1} = f(x_{n-1}), \ y_n = f(b).$

Jan 9-1:50 PM

The Trapezoidal Rule

To approximate $\int_a^b f(x)dx$, use

$$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n),$$

where $[a,b]$ is partitioned into n subintervals of equal length $h = (b-a)/n$.

Equivalently, $T = \dfrac{\text{LRAM}_n + \text{RRAM}_n}{2}$,

where LRAM_n and RRAM_n are the Rienamm sums using the left and right endpoints, respectively, for f for the partition.

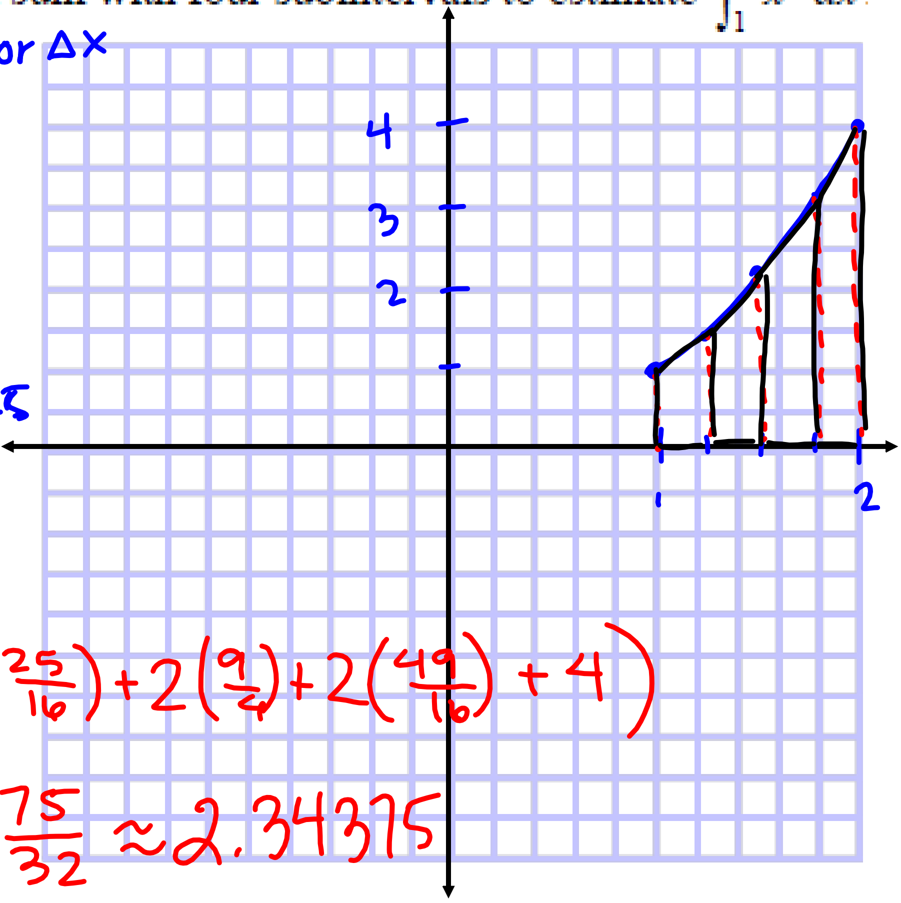
Jan 9-1:53 PM

Examples

1A. Use a trapezoidal sum with four subintervals to estimate $\int_1^2 x^2\,dx$.

$h = \frac{2-1}{4} = \frac{1}{4}$ or Δx

x	y
1	1
$\frac{5}{4}$	$\frac{25}{16} \approx 1.5$
$3/2$	$9/4 \approx 2.25$
$7/4$	$49/16 \approx 3.1$
2	4



$T = \frac{1}{2} \cdot \frac{1}{4}\left(1 + 2\left(\frac{25}{16}\right) + 2\left(\frac{9}{4}\right) + 2\left(\frac{49}{16}\right) + 4\right)$

$T = \frac{1}{8}\left(\frac{75}{4}\right) = \frac{75}{32} \approx 2.34375$

B. Evaluate $\int_1^2 x^2\,dx$ without a calculator.

$\int_1^2 x^2\,dx = \frac{x^3}{3}\Big]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \approx 2.\overline{3}$

C. How does your estimate in part A compare with the exact value found in part B?

Part A is just a little more than part B.

* when concave up, $f''(x) > 0$ the trapezoidal rule is an overestimate.

* when concave down, $f''(x) < 0$ trap. rule is an underestimate

* when $f''(x)$ is constant, neither concave ↑ or concave ↓, the trap. rule is exact.

More Examples...

2. An observer measures the outside temperature, in degrees Fahrenheit, every hour from noon (hour 0) to midnight (hour 12) with the following results:

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°F)	63	65	66	68	70	69	68	68	65	64	62	58	55

A. Using $T(h)$ to represent the temperature function, some values of which are shown in the table above, write an expression that would allow you to find the average temperature over the 12-hour period.

$$av(T) = \frac{1}{12-0}\int_0^{12} T(h)\,dh$$

B. Use a trapezoidal sum with 6 subintervals of equal length to estimate the average temperature in the hours between noon and midnight.

$$h = \frac{12-0}{6} = \frac{12}{6} = 2$$

$$T = \frac{1}{2}\cdot 2\left[63 + 2\cdot 66 + 2\cdot 70 + 2\cdot 68 + 2\cdot 65 + 2\cdot 62 + 55\right]$$

$$T = 1[780] \div 780$$

Homework

6.5: pg.246-7 #1, 2, 4, 7, 9-12