Questions on Lesson 6.4?

Factor the following to check yourself to

make sure you understand!

Step#1: Factor out GCF if there is one.

1. Completely factor $25x^2 - 10x - 24$ over the set of real numbers.

let $z = 5x \xrightarrow{so} (5x)^2 - 2(5x) - 24$

substitution pg. 462

$z^2 - 2z - 24$

$(z+4)(z-6)$

$(5x+4)(5x-6)$

Factor by grouping:

$25x^2 - 10x - 24$

$(25x^2 - 30x)(+20x - 24)$

$5x(5x-6) + 4(5x-6)$

$(5x-6)(5x+4)$

$25 \cdot -24 = -600$

$20 \cdot -30 = -600$

$(5x + \frac{-12}{8})(5x + \frac{+2}{3})$

$(x \quad)(25x \quad)$

2. Completely factor $x^3 - 4x^2 - 9x + 36$ over the set of real numbers.

3. Completely factor $x^4 - 25x^2 + 144$ over the set of real numbers.

$(x^2 - 16)(x^2 - 9)$

$(x+4)(x-4)(x+3)(x-3)$

4. Completely factor $27x^3 - 18x^2 + 3x$ over the set of real numbers.

$3x(9x^2 - 6x + 1) = 3x(3x-1)^2$

$3x((3x)^2 - 2(3x)(1) + (1)^2)$

$a = 3x$

$b = 1$

$2ab = 2 \cdot 3x \cdot 1 = 6x$

5. Completely factor $16x^3 + 54$ over the set of real numbers.

$2(8x^3 + 27)$

$a = 2x$

$b = 3$

$2(2x+3)((2x)^2 - (2x)(3) + (3)^2)$

$\boxed{2(2x+3)(4x^2 - 6x + 9)}$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

6. Completely factor $7x^4 - 56x$ over the set of real numbers.

$7(x^4 - 8x) = 7x(x^3 - 8)$

$a = x$

$b = 2$

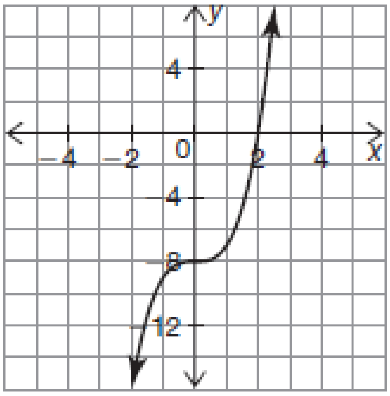
$= \boxed{7x(x-2)(x^2 + 2x + 4)}$

Feb 27-8:47 AM

pg.465 in your book.

1. Factor each polynomial function over the set of real numbers.

   a. $f(x) = x^3 - 8$



pg.466 in your book.

You may have noticed that all the terms in the polynomials from Question 1 are perfect cubes. You can rewrite the expression $x^3 - 8$ as $(x)^3 - (2)^3$, and $x^3 + 27$ as $(x)^3 + (3)^3$. When you factor sums and differences of cubes, there is a special factoring formula you can use, which is similar to the difference of squares for quadratics.

To determine the formula for the difference of cubes, generalize the difference of cubes as $a^3 - b^3$.

To determine the factor formula for the difference of cubes, factor out $(a - b)$ by considering $(a^3 - b^3) \div (a - b)$.

$$\begin{array}{r|l} & a^2 + ab + b^2 \\ a - b & a^3 + 0 + 0 - b^3 \\ & \underline{a^3 - a^2b} \\ & a^2b + 0 \\ & \underline{a^2b - ab^2} \\ & ab^2 - b^3 \\ & \underline{ab^2 - b^3} \\ & 0 \end{array}$$

$8x^3 - 27$

$a = 2x$

$b = 3$

$8x^3 - 27 = (2x-3)((2x)^2 + (2x)(3) + (3)^2)$

$(2x-3)(4x^2 + 6x + 9)$

Therefore, the difference of cubes can be rewritten in factored form:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$8x^3 + 27$

$a = 2x$

$b = {}^-3$

Finishing up 6.4
pg.467 in your book

Remember that you can factor a binomial that has perfect square *a*- and *c*-values and no middle value using the difference of squares.

You can use the difference of squares when you have a binomial of the form $a^2 - b^2$.

The binomial $\boxed{a^2 - b^2 = (a + b)(a - b).}$

3. Use the difference of squares to factor each binomial over the set of real numbers.

a. $x^2 - 64$

$a = x$

$b = 8$

$(x-8)(x+8)$

b. $x^4 - 16$

pg.468 in your book

Another special form of polynomial is the perfect square trinomials. Perfect square trinomials occur when the polynomial is a trinomial, and where the first and last terms are perfect squares and the middle term is equivalent to 2 times the product of the first and last term's square root.

Factoring a perfect square trinomial can occur in two forms:

$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 + 2ab + b^2 = (a + b)^2$$

4. Determine which of the polynomial expression(s) is a perfect square trinomial and write it as a sum or difference of squares. If it is not a perfect square trinomial, explain why.

a. $\cancel{x^4 + 14x^2 - 49}$

$x^4 - 14x^2 + 49$

$(x^2 - 7)^2$

$a = x^2$

$b = 7$

$2ab = 2 \cdot x^2 \cdot 7$

$= 14x^2$

b. $16x^2 - 40x + 100$

Getting to the Root of It All

Rational Root Theorem

6.5

pg.471-472 in your book

Consider the product and sum of each set of roots.

Polynomial	Roots	Product of Roots	Sum of Roots
$x^2 + 4x - 1 = 0$	$-2 \pm \sqrt{5}$	-1	-4
$x^3 + 2x^2 - 5x - 6 = 0$	$-1, 2, -3$	6	-2
$2x^3 + 5x^2 - 8x - 20 = 0$	$\pm 2, -\frac{5}{2}$	10	$-\frac{5}{2}$
$4x^3 - 3x^2 + 4x - 3 = 0$	$\pm i, \frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$36x^3 + 24x^2 - 43x + 86 = 0$	$\frac{2}{3} \pm \frac{\sqrt{3}}{2} i, -2$	$-\frac{43}{18}$	$-\frac{2}{3}$
$4x^4 - 12x^3 + 13x^2 - 2x - 6 = 0$	$1 \pm i, -\frac{1}{2}, \frac{3}{2}$	$-\frac{3}{2}$	3

1. Compare the sums of the roots to the first two coefficients of each polynomial equation. What conclusion can you draw?

2. Compare the products of the roots to the first and last coefficients of each odd degree polynomial equation. What conclusion can you draw?

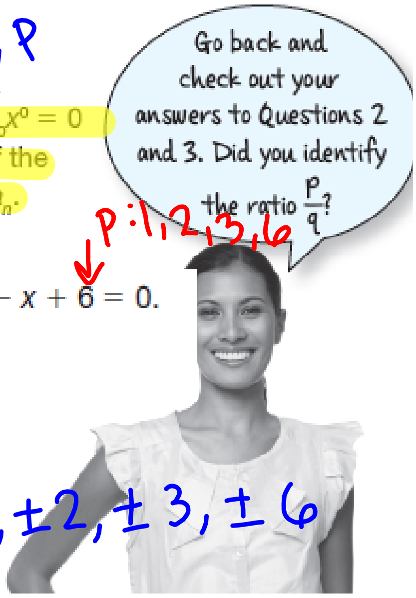
3. Compare the products of the roots to the first and last coefficients of each even degree polynomial equation. What conclusion can you draw?

pg.473 in your book

Up until this point, in order to completely factor a polynomial with a degree higher than 2, you needed to know one of the factors or roots. Whether that was given to you, taken from a table, or graph and verified by the Factor Theorem, you started out with one factor or root. What if you are not given any factors or roots? Should you start randomly choosing numbers and testing them to see if they divide evenly into the polynomial? This is a situation when the *Rational Root Theorem* becomes useful.

q p

The **Rational Root Theorem** states that a rational root of a polynomial equation $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0x^0 = 0$ with integer coefficients is of the form $\frac{p}{q}$, where p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.



pg.474 in your book

q : 1

p : 1, 2, 3, 6

5. Complete each step to factor and solve $x^4 + x^3 - 7x^2 - x + 6 = 0$.

a. Determine all the possible rational roots.

$\pm\left\{\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{6}{1}\right\}$

$\pm\{1, 2, 3, 6\}$ -OR- $\pm1, \pm2, \pm3, \pm6$

b. Use synthetic division to determine which of the possible roots are actual roots.

$(x-1)$

1	1	1	-7	-1	6
		1	2	-5	-6
	1	2	-5	-6	$\boxed{0}$

1 is a root

c. Rewrite the polynomial as a product of its quotient and linear factor.

$(x-1)(x^3+2x^2-5x-6)$

$(x+1)$

-1	1	2	-5	-6
		-1	-1	6
	1	1	-6	0

d. Repeat steps $a-c$ for the cubic expression.

$\pm\{1, 2, 3, 6\}$

$(x-1)$

1	1	2	-5	-6
	↓	1	3	-2
	1	3	-2	-8

e. Factor completely and solve.

$(x-1)(x+1)(x^2+x-6)$

$(x-1)(x+1)(x+3)(x-2) = 0$

$x = 1, -1, -3, 2$

pgs.475-478 in your book are **homework**! What's below is not in your book, so work these problems out on a piece of paper in your binder!

1. Use the Rational Root Theorem to solve $x^3 - 2x^2 - 5x - 2 = 0$.

3. Use the Rational Root Theorem to solve $2x^4 - 9x^3 + 11x^2 - 9x + 9 = 0$.

2. Use the Rational Root Theorem to solve $\overset{q}{2}x^3 - 2x^2 - 6x - \overset{p}{18} = 0$.

All of the p's: 1, 2, 3, 6, 9, 18

All of the q's: 1, 2

$$\frac{p}{q} \rightarrow \pm\left\{\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{6}{1}, \frac{9}{1}, \frac{18}{1}, \frac{1}{2}, \cancel{\frac{2}{2}}, \frac{3}{2}, \cancel{\frac{6}{2}}, \frac{9}{2}, \cancel{\frac{18}{2}}\right\}$$

SIMPLIFY

$$\pm\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, 3, \frac{9}{2}, 6, 9, 18\right\}$$

Oct 9-8:14 AM

Not in your book - these are for additional practice

$$f(x) = 5x^3 - 3x^2 + 5x - 3$$

$$f(x) = 3x^4 - 25x^2 - 18$$

$$f(x) = 3x^3 - x^2 - 3x + 1$$

$$f(x) = 4x^3 - 3x^2 + 4x - 3$$

Homework
Finish Lesson 6.5