Questions on 6.3 HW?

32, 33, 20

avg

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

32 $y = \frac{1}{x}$, $[e, 2e]$

$$\frac{1}{2e-e}\int_e^{2e}\left(\frac{1}{x}\right)dx = \frac{1}{e}(\ln 2e - \ln e)$$

$$= \frac{1}{e}\left(\ln\left(\frac{2e}{e}\right)\right) = \frac{1}{e}(\ln 2) = \frac{\ln 2}{e}$$

OR

33 $y = \sec^2 x$ $\left[0, \frac{\pi}{4}\right]$

antiderivative

$\tan x$

20 $$\int_0^{\pi/2} \cos x\,dx = \sin(\pi/2) - \sin(0) = 1 - 0 = 1$$

6.4 Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus - Part 1

If f is continuous on $[a,b]$, then the function $F(x)=\int_a^x f(t)dt$

has a derivative at every point x in $[a,b]$, and

$$\frac{dF}{dt}=\underbrace{\frac{d}{dx}\int_a^x f(t)dt=f(x)}.$$

Or more simply stated...

$$\frac{d}{dx}\int_a^x f(t)dt=f(x)$$

Every continuous function f is the derivative of some other function.

Every continuous function has an antiderivative.

The processes of integration and differentiation are inverses of one another.

Example

Find $\dfrac{d}{dx}\int_{\pi}^{x} \sin t\,dt$.

$$\frac{d}{dx}\int_{\pi}^{x} \sin t\,dt = \sin x$$

Answer

$$\frac{d}{dx}\int_{\pi}^{x} \sin t\,dt = \sin x$$

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More Examples

Evaluate:

$$\frac{d}{dx}\int_4^x t^2\,dt = x^2$$

$$\frac{d}{dx}\int_\pi^x \ln t\,dt = \ln x$$

$$\frac{d}{dx}\int_0^x e^{4t^2-1}\,dt = e^{4x^2-1}$$

$$\frac{d}{dx}\int_x^5 \left(9t^3-\cos t\right)dt =$$

$$-\frac{d}{dx}\int_5^x (9t^3-\cos t)\,dt =$$

$$-(9x^3-\cos x)$$

$$\frac{d}{dx}\int_{-6}^{\overset{u}{\sin x}} \left(t^3-4t\right)dt =$$

$u=\sin x$

$du=\cos x$

$$\frac{d}{du}\int_{-6}^{u}(t^3-4t)\,dt\cdot\frac{du}{dx}=$$

$$u^3-4u\cdot\frac{du}{dx}$$

$$\boxed{(\sin^3 x-4\sin x)\cos x}$$

$u=x^3-3x$

$du=3x^2-3$

$$\frac{d}{dx}\int_0^{x^3-3x}\frac{1}{1-t^2}\,dt=$$

$$\frac{1}{1-(x^3-3x)^2}\cdot(3x^2-3)$$

$$=\boxed{\frac{3x^2-3}{1-(x^3-3x)^2}}$$

$$\frac{d}{dx}\int_x^{x^2}\frac{1}{t}\,dt=$$

$$\int_0^{x^2}\cdots + -\int_0^{x}\cdots$$

$$\frac{d}{dx}\int_0^{x^2}\frac{1}{t}\,dt-\frac{d}{dx}\int_0^{x}\frac{1}{t}\,dt=$$

$$\frac{1}{x^2}\cdot 2x-\frac{1}{x}=$$

$$\frac{2x}{x^2}-\frac{1}{x}=$$

$$\frac{2}{x}-\frac{1}{x}=\frac{1}{x}$$

$$\frac{d}{dx}\int_{2x}^{x^2}(\tan t+5)\,dt=$$

$$\int_0^{x^2}\cdots + -\int_0^{2x}\cdots$$

$$\frac{d}{dx}\int_0^{x^2}(\tan t+5)\,dt-\int_0^{2x}(\tan t+5)\,dt$$

$$(\tan(x^2)+5)(2x)-(\tan(2x)+5)(2)$$

$$\boxed{2x\tan x^2+10x-2\tan 2x-10}$$

The Fundamental Theorem of Calculus - part 2

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem**.

And...

$$\int_a^b f(x)dx = F(b) - F(a)$$

Any definite integral of any continuous function f can be calculated without taking limits, without calculating Riemann sums, and often without effort — so long as an antiderivative of f can be found.

Examples

Evaluate.

$$\int_{-1}^{3} (x^3+1)\,dx = \left[\frac{x^4}{4}+x\right]_{-1}^{3} =$$

$$\left(\frac{3^4}{4}+3\right)-\left(\frac{(-1)^4}{4}-1\right)=$$

$$\frac{93}{4}-\left(-\frac{3}{4}\right)=\frac{96}{4}=\boxed{24}$$

$$\int_{0}^{5} e^x\,dx = \left[e^x\right]_0^5 =$$

$$e^5-e^0=\boxed{e^5-1}$$

$$\int_{1}^{4} \frac{1}{x}\,dx = \left[\ln x\right]_1^4$$

$$\ln 4 - \cancel{\ln 1}^{\,0} = \boxed{\ln 4}$$

$$\int_{0}^{\frac{\pi}{4}} (\cos x - 1)\,dx =$$

$$\left[\sin x - x\right]_0^{\pi/4}$$

$$\left(\sin\left(\frac{\pi}{4}\right)-\frac{\pi}{4}\right)-\cancel{(\sin 0 - 0)}^{\,0}$$

$$\sin\left(\frac{\pi}{4}\right)-\frac{\pi}{4}$$

Homework

6.4: pg.306-7 #2-18 even, 27-32, 45, 48