Questions on Lesson 6.3?

Do you know how to determine if something is a factor of a polynomial?

Try this: Is (x - 7) a factor of

$$\left(x^5 - 49x^3 - 3x^2 + 28x - 53\right)?$$

How do you know?

$f(7) = (7)^5 - 49(7)^3 - 3(7)^2 + 28(7) - 53$

$f(7) = -4$ so x-7 is not a factor

$$\begin{array}{r|rrrrrr} 7 & 1 & 0 & -49 & -3 & 28 & -53 \\ & \downarrow & 7 & 49 & 0 & -21 & 49 \\ \hline & 1 & 7 & 0 & -3 & 7 & -4 \end{array}$$

6.4 Break It Down
Factoring Higher Order Polynomials

pg.459-460 in your book

In this lesson, you will explore different methods of factoring. To begin factoring any polynomial, always look for a greatest common factor (GCF). You can factor out the greatest common factor of the polynomial, and then factor what remains.

Remember, a greatest common factor can be a variable, constant, or both.

1. Ping and Shalisha each attempt to factor $3x^3 + 12x^2 - 36x$ by factoring out the greatest common factor.

Ping's Work

$3x^3 + 12x^2 - 36x$

$3x(x^2 + 4x - 12)$

Shalisha's Work

$3X^3 + 12X^2 - 36X$

$3(X^3 + 4X^2 - 12X)$

Analyze each student's work. Determine which student is correct and explain the inaccuracy in the other student's work.

Ping factored out the GCF, Shalisha's math is correct.

2. If possible, completely factor the expression that Ping and Shalisha started.

$3x(x^2+4x-12)$

$3x(x+6)(x-2)$

$0 = 3x(x+6)(x-2)$

$\frac{3x}{3} = \frac{0}{3}$ & $x + 6 = 0$ & $x - 2 = 0$

$-6 \quad -6 \qquad +2 \quad +2$

$x = 0 \qquad x = -6 \qquad x = 2$

$x = 0, -6, 2$

$r = 0, -6, 2$

Oct 9-8:12 AM

pg.461 in your book

3. Factor each expression over the set of real numbers. Remember to look for a greatest common factor first. Then, use the factors to sketch the graph of each polynomial.

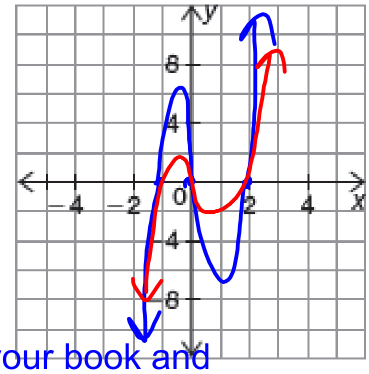
odd +

a. $3x^3 - 3x^2 - 6x$

$3x(x^2 - x - 2)$

$3x(x-2)(x+1)$

$x = 0, 2, -1$



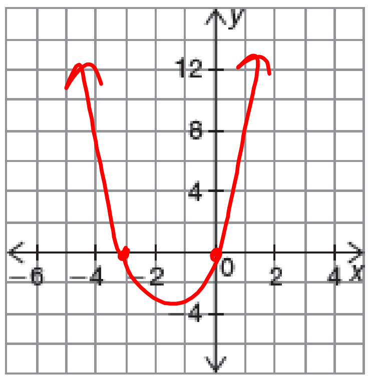
finish b, c d, and e on pg.461 in your book and 4-5 on pg.462

even +

c. $2x^2 + 6x$

$2x(x+3)$

$x = 0, -3$



4. Analyze the factored form and the corresponding graphs in Question 3. What do the graphs in part (a) through part (c) have in common that the graphs of part (d) and part (e) do not? Explain your reasoning.

d&e's GCF doesn't have a variable

5. Write a statement about the graphs of all polynomials that have a monomial GCF that contains a variable.

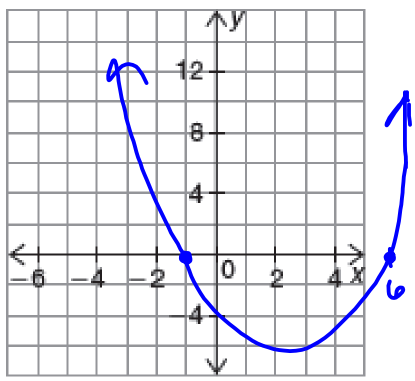
We will have an x-intercept of 0.

~~c. $2x^2 + 6x$~~

e) $10x^2 - 50x - 60$

$10(x^2 - 5x - 6)$

$10(x-6)(x+1)$



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pg.462 in your book

PROBLEM 2 Continue Parsing

Some polynomials in quadratic form may have common factors in some of the terms, but not all terms. In this case, it may be helpful to write the terms as a product of 2 terms. You can then substitute the common term with a variable, z, and factor as you would any polynomial in quadratic form. This method of factoring is called *chunking*.

You can use chunking to factor $9x^2 + 21x + 10$.

Notice that the first and second terms both contain the common factor, $3x$.

$9x^2 + 21x + 10 = (3x)^2 + 7(3x) + 10$	Rewrite terms as a product of common factors.
$= z^2 + 7z + 10$	Let $z = 3x$.
$= (z + 5)(z + 2)$	Factor the quadratic.
$= (3x + 5)(3x + 2)$	Substitute $3x$ for z.

The factored form of $9x^2 + 21x + 10$ is $(3x + 5)(3x + 2)$.

1. Use chunking to factor $49x^2 + 35x + 6$.

$z = 7x$

$(7x)^2 + 5(7x) + 6$

$z^2 + 5z + 6$

$(z+3)(z+2)$

$(7x+3)(7x+2)$

pg.463 in your book

Using a similar method of factoring, you may notice, in polynomials with 4 terms, that although not all terms share a common factor, pairs of terms might share a common factor. In this situation, you can *factor by grouping*.

3. Colt factors the polynomial expression $x^3 + 3x^2 - x - 3$.

$(x+3)(x^2-1)$

$x^2(x+3) - 1(x+3)$

Colt

$(x^3 + 3x^2)(- x - 3)$

$x^2(x + 3) - 1(x + 3)$

$(x + 3)(x^2 - 1)$

$(x + 3)(x + 1)(x - 1)$

difference of squares

4. Use factor by grouping to factor the polynomial expression $(x^3 + 7x^2)(- 4x - 28)$.

$x^2(x + 7) - 4(x + 7)$

$(x+7)(x^2-4)$

$(x+7)(x+2)(x-2)$

pg.464 in your book. $a+bi$

Recall that the Fundamental Theorem of Algebra states that any polynomial equation of degree *n* must have exactly *n* complex roots or solutions. Also, the Fundamental Theorem of Algebra states that every polynomial function of degree *n* must have exactly *n* complex zeros. $-2=-2+0i$ any real # can be written as $a+bi$

This implies that any polynomial function of degree *n* must have exactly *n* complex factors:

$f(x) = (x - r_1)(x - r_2) \ldots (x - r_n)$ where $r \in$ {*complex numbers*}.

Some 4th degree polynomials, written as a trinomial, look very similar to quadratics as they have the same form, $ax^4 + bx^2 + c$. When this is the case, the polynomial may be factored using the same methods you would use to factor a quadratic. This is called *factoring by using quadratic form*.

Factor the quartic polynomial by using quadratic form.

$x^4 - 29x^2 + 100$	• Determine whether you can factor the given trinomial into 2 factors.
$(x^2 - 4)(x^2 - 25)$	• Determine if you can continue to factor each binomial.
$(x - 2)(x + 2)(x - 5)(x + 5)$	

pg.465 in your book.

6. Factor each polynomial expression over the set of complex numbers.

a. $(x^4 - 4x^3)(-x^2 + 4x)$

$x^3(x-4)-x(x-4)$

$(x-4)(x^3-x)$

$(x-4)(x)(x^2-1)$

$\boxed{(x-4)(x)(x+1)(x-1)}$

b. $x^4 - 10x^2 + 9$

$(x^2-9)(x^2-1)$

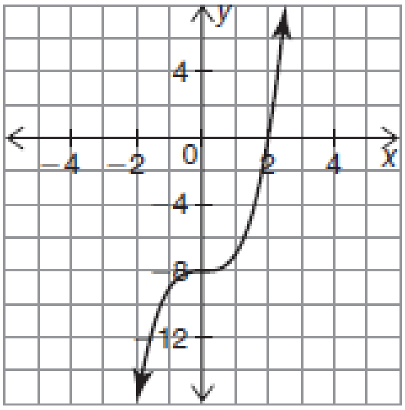
$\boxed{(x+3)(x-3)(x+1)(x-1)}$

Nov 10-8:45 AM

pg.465 in your book.

1. Factor each polynomial function over the set of real numbers.

 a. $f(x) = x^3 - 8$



pg.466 in your book.

You may have noticed that all the terms in the polynomials from Question 1 are perfect cubes. You can rewrite the expression $x^3 - 8$ as $(x)^3 - (2)^3$, and $x^3 + 27$ as $(x)^3 + (3)^3$. When you factor sums and differences of cubes, there is a special factoring formula you can use, which is similar to the difference of squares for quadratics.

To determine the formula for the difference of cubes, generalize the difference of cubes as $a^3 - b^3$.

To determine the factor formula for the difference of cubes, factor out $(a - b)$ by considering $(a^3 - b^3) \div (a - b)$.

$$\begin{array}{rl} & a^2 + ab + b^2 \\ a - b \,\big) & \overline{a^3 + 0 + 0 - b^3} \\ & \underline{a^3 - a^2b} \\ & \quad a^2b + 0 \\ & \quad \underline{a^2b - ab^2} \\ & \qquad ab^2 - b^3 \\ & \qquad \underline{ab^2 - b^3} \\ & \qquad\qquad 0 \end{array}$$

Therefore, the difference of cubes can be rewritten in factored form:
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Nov 19-8:31 AM

pg.467 in your book

Remember that you can factor a binomial that has perfect square a- and c-values and no middle value using the difference of squares.

You can use the difference of squares when you have a binomial of the form $a^2 - b^2$.

The binomial $a^2 - b^2 = (a + b)(a - b)$.

3. Use the difference of squares to factor each binomial over the set of real numbers.

   a. $x^2 - 64$

   b. $x^4 - 16$

pg.468 in your book

Another special form of polynomial is the perfect square trinomials. Perfect square trinomials occur when the polynomial is a trinomial, and where the first and last terms are perfect squares and the middle term is equivalent to 2 times the product of the first and last term's square root.

Factoring a perfect square trinomial can occur in two forms:

$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 + 2ab + b^2 = (a + b)^2$$

4. Determine which of the polynomial expression(s) is a perfect square trinomial and write it as a sum or difference of squares. If it is not a perfect square trinomial, explain why.

   a. $x^4 + 14x^2 - 49$

   b. $16x^2 - 40x + 100$

not in your book.

1. Completely factor $25x^2 - 10x - 24$ over the set of real numbers.

2. Completely factor $x^3 - 4x^2 - 9x + 36$ over the set of real numbers.

3. Completely factor $x^4 - 25x^2 + 144$ over the set of real numbers.

4. Completely factor $27x^3 - 18x^2 + 3x$ over the set of real numbers.

5. Completely factor $16x^3 + 54$ over the set of real numbers.

6. Completely factor $7x^4 - 56x$ over the set of real numbers.

7. Consider the function $f(x) = 4x^3 - 12x^2 + 9x - 2$.

   a. Complete the table of values.

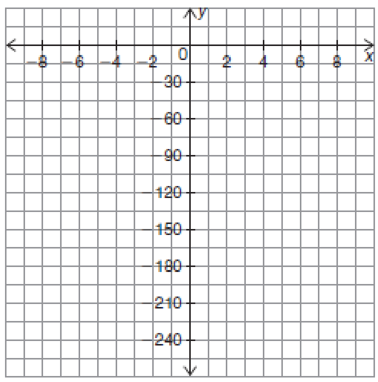
x	-2	-1	0	1	2
$f(x)$					

   b. Completely factor $f(x)$ over the set of real numbers.

8. Consider the function $h(x) = x^4 + 3x^3 - 17x^2 + 3x - 18$.

   a. Describe how to determine one of the factors of $h(x)$ in order to begin factoring the function.

   b. Completely factor $h(x)$ over the set of real numbers.



Homework
Finish Lesson 6.4