Questions on Lesson 6.2?

We'll have a polynomial division problem for our quiz today, so make sure you know how to divide polynomials using either long division or synthetic division.

(12)

$$\begin{array}{r|rrrr} -2 & 5 & 10 & 0 & -4 \\ & \downarrow & -10 & 0 & 0 \\ \hline & 5 & 0 & 0 & \boxed{-4} \end{array}$$

$p(x)=5x^3+10x^2-4$

$r+2$

$p(-2)=5(-2)^3+10(-2)^2-4$

$p(-2)=\cancel{-40}+\cancel{40}-4$

$p(-2)=-4$

$$\boxed{5r^2-\frac{4}{r+2}}$$

(4)

$$\begin{array}{r} a^3+6a^2-3a+8 \\ 3a-7\overline{)\cancel{3a^4}+11a^3-51a^2+45a-59} \\ -(\cancel{3a^4}-7a^3) \downarrow \quad \downarrow \quad \downarrow \\ \hline \cancel{18a^3}-51a^2+45a-59 \\ -(\cancel{18a^3}-42a^2) \downarrow \quad \downarrow \\ \hline -\cancel{9a^2}+45a-59 \\ -(-\cancel{9a^2}+21a) \downarrow \\ \hline \cancel{24a}-59 \\ -(\cancel{24a}-56) \\ \hline \boxed{-3} \end{array}$$

Answer:

$a^3+6a^2-3a+8-\frac{3}{3a-7}$

Polynomial Division Content Mastery Quiz

Use either long division **or** synthetic division

to divide the following.

$$\left(a^3 + a^2 - 34a + 51\right) \div \left(a + 7\right)$$

6.3 The Factors of Life

The Factor Theorem and Remainder Theorem

pg.451-452 in your book

You learned that the process of dividing polynomials is similar to the process of dividing integers. Sometimes when you divide two integers there is a remainder, and sometimes there is not a remainder. What does each case mean? In this lesson, you will investigate what the remainder means in terms of polynomial division.

Remember from your experiences with division that:

$\frac{452}{10} = 45 + \frac{2}{10}$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

or

$452 = 10(45) + 2$

$$\text{dividend} = (\text{divisor})(\text{quotient}) + \text{remainder}.$$

It follows that any polynomial, $p(x)$, can be written in the form:

$$\frac{p(x)}{\text{linear factor}} = \text{quotient} + \frac{\text{remainder}}{\text{linear factor}}$$

or

$$p(x) = (\text{linear factor})(\text{quotient}) + \text{remainder}.$$

Generally, the linear factor is written in the form $(x - r)$, the quotient is represented by $q(x)$, and the remainder is represented by R, meaning:

$$\boxed{p(x) = (x - r)q(x) + R.}$$

$\frac{n^3 - 16n^2 + 57n + 57}{n-9} = n^2 - 7n - 6 + \frac{3}{n-9}$

$n^3 - 16n^2 + 57n + 57 = (n-9)(n^2 - 7n - 6) + 3$

Oct 9-8:12 AM

pg.452 in your book

1. Given $p(x) = x^3 + 8x - 2$ and $\frac{p(x)}{(x-3)} = x^2 + 3x + 17$ R 49.

a. Verify $p(x) = (x - r)q(x) + R$.

$x^3+8x-2=(x-3)(x^2+3x+17)+49$

$x^3+8x-2=x^3+\cancel{3x^2}+17x-\cancel{3x^2}-9x-51+49$

$x^3+8x-2=x^3+8x-2$

b. Given $x - 3$ is a linear factor of $p(x)$, evaluate $p(3)$.

$P(x)=x^3+8x-2$

$P(3)=3^3+8(3)-2$

$27+24-2=49$

2. Given $p(x) = (x - r)q(x) + R$, calculate $p(r)$.

$(r-r)q(x)+R$

$0(q(x))+R$

$=R$

3. Explain why $p(r)$, where $(x - r)$ is a linear factor, will always equal the remainder R, regardless of the quotient.

pg.453 in your book

4. What conclusion can you make about any polynomial evaluated at r?

We will always get the remainder, R.

The **Remainder Theorem** states that when any polynomial equation or function, $f(x)$, is divided by a linear factor $(x - r)$, the remainder is $R = f(r)$, or the value of the equation or function when $x = r$.

5 $p(x) = x^3 + 6x^2 + 5x - 12$ $\frac{p(x)}{(x-2)} = x^2 + 8x + 21$ R30

Paloma,
$p(2) = 30$

6. The function, $f(x) = 4x^2 + 2x + 9$ generates the same remainder when divided by $(x - r)$ and $(x - 2r)$, where r is not equal to 0. Calculate the value(s) of r.

$f(r) = 4r^2 + 2r + 9$ $f(2r) = 4(2r)^2 + 2(2r) + 9$

$f(2r) = 16r^2 + 4r + 9$

$f(r) = f(2r) = R$

$$\cancel{4r^2} + \cancel{2r} + \cancel{9} = 16r^2 + 4r + \cancel{9}$$
$$-\cancel{4r^2} - \cancel{2r} - \cancel{9} \quad -4r^2 - 2r - \cancel{9}$$

$0 = 12r^2 + 2r$

$0 = 2r(6r + 1)$

$\frac{2r}{2} = \frac{0}{2}$ and $6r + \cancel{1} = 0$
$-\cancel{1} \quad -1$

~~$r = 0$~~ $\frac{6r}{6} = \frac{-1}{6}$

$\boxed{r = -1/6}$

pg.454 in your book.

Consider the factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.

Notice that when you divide 24 by any of its factors the remainder is 0. This same principle holds true for polynomial division.

The **Factor Theorem** states that a polynomial has a linear polynomial as a factor if and only if the remainder is zero; or, in other words, $f(x)$ has $(x - r)$ as a factor if and only if $f(r) = 0$.

pg.455 in your book.

You can continue to factor the polynomial $f(x) = x^3 - 10x^2 + 11x + 70$.

From Haley and Lillian's work, you know that $f(x) = (x - 7)(x^2 - 3x - 10)$.

The quadratic factor can also be factored.

$$f(x) = (x - 7)(x^2 - 3x - 10)$$

$$f(x) = (x - 7)(x + 2)(x - 5)$$

$x = 7, -2, 5$

$r = 7, -2, 5$

2. Use the Factor Theorem to prove each factor shown in the worked example is correct.

$f(7) = 7^3 - 10(7)^2 + 11(7) + 70$

$f(7) = 0$

$f(-2) = (-2)^3 - 10(-2)^2 + 11(-2) + 70$

$f(-2)$

$f(5) = (5)^3 - 10(5)^2 + 11(5) + 70$

$f(5) =$

3. What other method(s) could you use to verify that the factors shown in the worked example are correct?

+ Graphing

+ Long ÷ or synthetic ÷

pg.456 in your book.

4. Use the Factor Theorem to prove that $f(x) = (x + 1 - 3i)(x + 1 + 3i)$ is the factored form of $f(x) = x^2 + 2x + 10$.

$x = -1+3i, -1-3i$
$r = -1+3i, -1-3i$

$f(-1+3i) = (-1+3i)^2 + 2(-1+3i) + 10$
$= (-1+3i)(-1+3i) + -2 + 6i + 10$
$= 1 - 3i - 3i + 9i^2 - 2 + 6i + 10$
$= 1 - \cancel{6i} - 9 - 2 + \cancel{6i} + 10$
$= -10 + 10$
$f(-1+3i) = 0$

$i = \sqrt{-1}$
$i^2 = -1$

$f(-1-3i) = (-1-3i)^2 + 2(-1-3i) + 10$

$\vdots$

5. Determine the unknown coefficient, a, in each function.

a. $f(x) = 2x^4 + x^3 - 14x^2 - ax - 6$ if $(x - 3)$ is a linear factor.

b. $f(x) = ax^4 + 25x^3 + 21x^2 - x - 3$ if $(x + 1)$ is a linear factor.

not in your book

1. Given: $\frac{f(x)}{(x-3)} = x^2 - 7x - 13\ R\ 25$.

   a. Determine $f(3)$ using the Remainder Theorem. Explain your reasoning.

   b. Determine $f(x)$.

   c. Determine whether $x - 8$ is a factor of $f(x)$. Explain your reasoning.

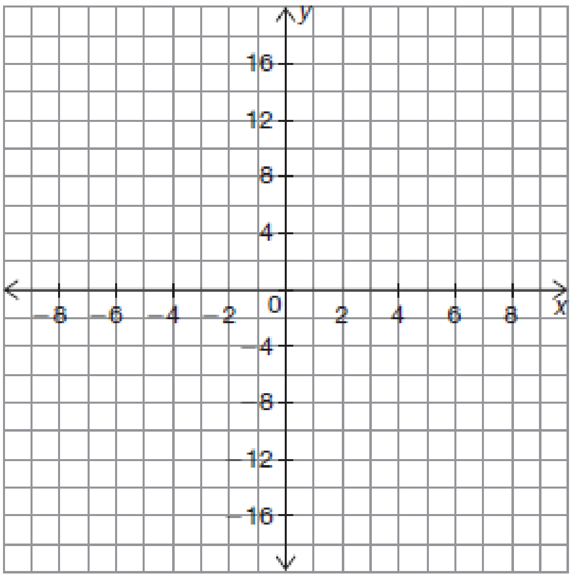
   d. Determine $f(8)$ using the Factor Theorem. Explain your reasoning.

   e. Completely factor $f(x)$.

Nov 19-8:32 AM

not in your book.

2. Given: $\frac{g(x)}{x-1} = x^2 + x - 8\ R - 8$, $\frac{g(x)}{x-2} = x^2 + 2x - 5\ R - 10$, and $\frac{g(x)}{x-3} = x^2 + 3x\ R\ 0$.
The function $g(x)$ is cubic and its graph is symmetric about the origin. Use the given information and the Remainder Theorem to sketch the graph of $g(x)$ on the coordinate plane provided.



3. The function $m(x) = 2x^2 + 6x - 7$ generates the same remainder when divided by $(x - a)$ and $(x - 2a)$ when $a \neq 0$. Calculate the value(s) of a and determine the corresponding factors.

Homework
Finish Lesson 6.3

Oct 21-8:47 AM