Questions on Lesson 6.2?

We'll have a polynomial division problem for our quiz today, so make sure you know how to divide polynomials using either long division or synthetic division.

$$\frac{8k^{5}}{8k} = k^4$$

$$\begin{array}{r} k^4 + 6k^3 + 4k^2 - 5k + 9 \\ 8k-6 \overline{) 8k^5 + 42k^4 - 4k^3 - 64k^2 + 102k - 49} \\ -(8k^5 - 6k^4) \\ \hline 48k^4 - 4k^3 - 64k^2 + 102k - 49 \\ -(48k^4 - 36k^3) \\ \hline 32k^3 - 64k^2 + 102k - 49 \\ -(32k^3 - 24k^2) \\ \hline -40k^2 + 102k - 49 \\ -(-40k^2 + 30k) \\ \hline 72k - 49 \\ -(72k - 54) \\ \hline \boxed{5} \end{array}$$

Answer: $k^4 + 6k^3 + 4k^2 - 5k + 9 + \frac{5}{8k-6}$

(13)

$$\begin{array}{r|rrrrr} -8 & 4 & 35 & 15 & -63 & 62 \\ & \downarrow & -32 & -24 & 72 & -72 \\ \hline & 4 & 3 & -9 & 9 & \boxed{-10} \end{array}$$

$$\boxed{4a^3 + 3a^2 - 9a + 9 - \frac{10}{a+8}}$$

$$4x^4 + 35x^3 + 15x^2 - 63x + 62$$

$$f(-8) = 4(-8)^4 + 15(-8)^2 - 63(-8) + 62$$

Feb 27-8:47 AM

Polynomial Division Content Mastery Quiz

Use either long division **or** synthetic division to divide the following.

$$\left(a^3 + a^2 - 34a + 51\right) \div \left(a + 7\right)$$

6.3 The Factors of Life

The Factor Theorem and Remainder Theorem

pg.451-452 in your book

You learned that the process of dividing polynomials is similar to the process of dividing integers. Sometimes when you divide two integers there is a remainder, and sometimes there is not a remainder. What does each case mean? In this lesson, you will investigate what the remainder means in terms of polynomial division.

Remember from your experiences with division that:

$$\frac{452}{10} = 45 + \frac{2}{10}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

or

$$452 = 10(45) + 2$$

$$\text{dividend} = (\text{divisor})(\text{quotient}) + \text{remainder}.$$

It follows that any polynomial, $p(x)$, can be written in the form:

$$\frac{p(x)}{\text{linear factor}} = \text{quotient} + \frac{\text{remainder}}{\text{linear factor}}$$

or

$$p(x) = (\text{linear factor})(\text{quotient}) + \text{remainder}.$$

Generally, the linear factor is written in the form $(x - r)$, the quotient is represented by $q(x)$, and the remainder is represented by R, meaning:

$$p(x) = (x - r)q(x) + R.$$

$$\frac{n^3-16n^2+57n+57}{n-9} = n^2-7n-6 + \frac{3}{n-9}$$

$$\underbrace{n^3-16n^2+57n+57}_{p(x)} = \underbrace{(n-9)}_{(x-r)}\underbrace{(n^2-7n-6)}_{q(x)} + \underbrace{3}_{R}$$

unfactored polynomial

pg.452 in your book

1. Given $p(x) = x^3 + 8x - 2$ and $\frac{p(x)}{(x-3)} = x^2 + 3x + 17$ R 49.

   a. Verify $p(x) = (x - r)q(x) + R$.

$x^3+8x-2 = (x-3)(x^2+3x+17)+49$

$= x^3+8x-51+49$

$= x^3+8x-2$ ✓

   b. Given $x - 3$ is a linear factor of $p(x)$, evaluate $p(3)$.

$P(3) = 3^3+8(3)-2$

$27+24-2 = 49$

2. Given $p(x) = (x - r)q(x) + R$, calculate $p(r)$.

$P(r) = (r-r)q(x)+R$

$= 0 \cdot q(x)+R$

$= 0+R = R$

3. Explain why $p(r)$, where $(x - r)$ is a linear factor, will always equal the remainder R, regardless of the quotient.

r-r will always = 0

0 · anything will = 0

left w/ R

pg.453 in your book

4. What conclusion can you make about any polynomial evaluated at *r*?

always left w/ R

The **Remainder Theorem** states that when any polynomial equation or function, $f(x)$, is divided by a linear factor $(x - r)$, the remainder is $R = f(r)$, or the value of the equation or function when $x = r$.

6. The function, $f(x) = 4x^2 + 2x + 9$ generates the same remainder when divided by $(x - r)$ and $(x - 2r)$, where r is not equal to 0. Calculate the value(s) of r.

$f(r) = 4r^2 + 2r + 9$

$f(2r) = 4(2r)^2 + 2(2r) + 9$

$f(2r) = 16r^2 + 4r + 9$

$f(r) = f(2r)$

$$\cancel{4r^2} + \cancel{2r} + \cancel{9} = 16r^2 + 4r + 9$$

$$-\cancel{4r^2} \quad -\cancel{2r} \quad -\cancel{9} \qquad -4r^2 \quad -2r \quad -9$$

$$0 = 12r^2 + 2r$$

$$0 = 2r(6r + 1)$$

$\frac{0}{2} = \frac{2r}{2}$ & $0 = 6r + \cancel{1}$

$-1 \qquad -\cancel{1}$

$\cancel{0 = r}$ $\qquad \frac{-1}{6} = \frac{6r}{6}$

$-\frac{1}{6} = r$

pg.454 in your book.

Consider the factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.

Notice that when you divide 24 by any of its factors the remainder is 0. This same principle holds true for polynomial division.

The **Factor Theorem** states that a polynomial has a linear polynomial as a factor if and only if the remainder is zero; or, in other words, $f(x)$ has $(x - r)$ as a factor if and only if $f(r) = 0$.

pg.455 in your book.

You can continue to factor the polynomial $f(x) = x^3 - 10x^2 + 11x + 70$.

From Haley and Lillian's work, you know that $f(x) = (x - 7)(x^2 - 3x - 10)$.

The quadratic factor can also be factored.

$$f(x) = (x - 7)(x^2 - 3x - 10)$$

$$f(x) = (x - 7)(x + 2)(x - 5)$$

$x = 7, -2, 5$

$r = 7, -2, 5$

2. Use the Factor Theorem to prove each factor shown in the worked example is correct.

$f(7) = 7^3 - 10(7)^2 + 11(7) + 70$

$f(7) = 0$

$f(-2) = (-2)^3 - 10(-2)^2 + 11(-2) + 70$

$f(-2) = 0$

$f(5) = (5)^3 - 10(5)^2 + 11(5) + 70$

$f(5) = 0$

3. What other method(s) could you use to verify that the factors shown in the worked example are correct?

pg.456 in your book.

4. Use the Factor Theorem to prove that $f(x) = (x + 1 - 3i)(x + 1 + 3i)$ is the factored form of $f(x) = x^2 + 2x + 10$.

$x = -1-3i, -1+3i$

$r = -1-3i, -1+3i$

$i = \sqrt{-1}$

$i^2 = -1$

$f(-1-3i) = (-1-3i)^2 + 2(-1-3i) + 10$

$f(-1-3i) = (-1-3i)(-1-3i) - 2 - 6i + 10$

$" \quad = 1 + \cancel{3i + 3i} + 9i^2 + 8 - \cancel{6i}$

$= 9 - 9$

$= 0$

$f(-1+3i) = (-1+3i)^2 + 2(-1+3i) + 10$

5. Determine the unknown coefficient, a, in each function.

a. $f(x) = 2x^4 + x^3 - 14x^2 - ax - 6$ if $(x - 3)$ is a linear factor.

b. $f(x) = ax^4 + 25x^3 + 21x^2 - x - 3$ if $(x + 1)$ is a linear factor.

not in your book

1. Given: $\dfrac{f(x)}{(x-3)} = x^2 - 7x - 13\ R\ 25$.

   a. Determine $f(3)$ using the Remainder Theorem. Explain your reasoning.

   b. Determine $f(x)$.

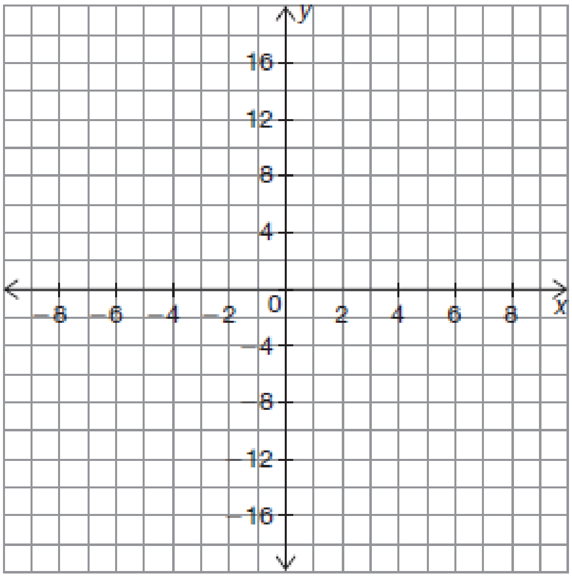
   c. Determine whether $x - 8$ is a factor of $f(x)$. Explain your reasoning.

   d. Determine $f(8)$ using the Factor Theorem. Explain your reasoning.

   e. Completely factor $f(x)$.

not in your book.

2. Given: $\dfrac{g(x)}{x-1} = x^2 + x - 8\ R - 8$, $\dfrac{g(x)}{x-2} = x^2 + 2x - 5\ R - 10$, and $\dfrac{g(x)}{x-3} = x^2 + 3x\ R\ 0$.
The function $g(x)$ is cubic and its graph is symmetric about the origin. Use the given information and the Remainder Theorem to sketch the graph of $g(x)$ on the coordinate plane provided.



3. The function $m(x) = 2x^2 + 6x - 7$ generates the same remainder when divided by $(x - a)$ and $(x - 2a)$ when $a \neq 0$. Calculate the value(s) of a and determine the corresponding factors.

Homework
Finish Lesson 6.3

Oct 21-8:47 AM