Friday, January 13 is the last day Ms. Hansen will accept any late/missing/extra credit work for 2nd quarter

-->This includes any test/quiz make ups.

Questions on 6.2 HW? Derivatives Quiz soon...

Quiz

6.3 Definite Integrals & Antiderivatives

Rules (pg.289)

Table 5.3 Rules for Definite Integrals

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ A definition

2. *Zero:* $\int_a^a f(x)\,dx = 0$ Also a definition

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ Any number k

   $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ $k = -1$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \le \int_a^b f(x)\,dx \le \max f \cdot (b - a).$$

7. *Domination:* $f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

$$f(x) \ge 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0 \qquad g = 0$$

EXAMPLES

Suppose f and g are integrable functions such that $\int_{-1}^{1} f(x)\,dx = 5$, $\int_{1}^{4} f(x)\,dx = -2$, and $\int_{-1}^{1} g(x)dx = -4$. Find each of the following:

a) $\int_{4}^{1} f(x)dx =$

$-\int_{1}^{4} f(x)\,dx = -(-2)$

$= 2$

b) $\int_{-1}^{4} f(x)dx =$

$\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx$

$= 5 + -2 = 3$

c) $\int_{0}^{1} f(x)dx$

not enough info

d) $\int_{-1}^{1} [2f(x)+3g(x)]dx$

$2\int_{-1}^{1} f(x)dx +$

$3\int_{-1}^{1} g(x)dx =$

$2(5)+3(-4)$

$10-12 = \boxed{-2}$

e) $\int_{-1}^{4} [f(x)+g(x)]dx$

not enough info

f) $\int_{-2}^{2} f(x)dx$

not enough info

Jan 9-1:53 PM

Average (Mean) Value

If f is integrable on $[a,b]$, its average (mean) value on $[a,b]$ is

$$avg(f)=\frac{1}{b-a}\int_a^b f(x)dx$$

Example

Find the average value of $f(x)=2-x^2$ on $[\overset{a}{0},\overset{b}{4}]$.

$$avg(f)=\frac{1}{4-0}\int_0^4(2-x^2)dx \xrightarrow{\text{NINT}} \frac{-10}{3}$$

Answer

$$avg(f)=\frac{1}{b-a}\int_a^b f(x)dx$$

$$=\frac{1}{4-0}\int_0^4\left(2-x^2\right)dx \quad \text{Use NINT to evaluate the integra}$$

$$=\frac{1}{4}\left(-\frac{40}{3}\right)$$

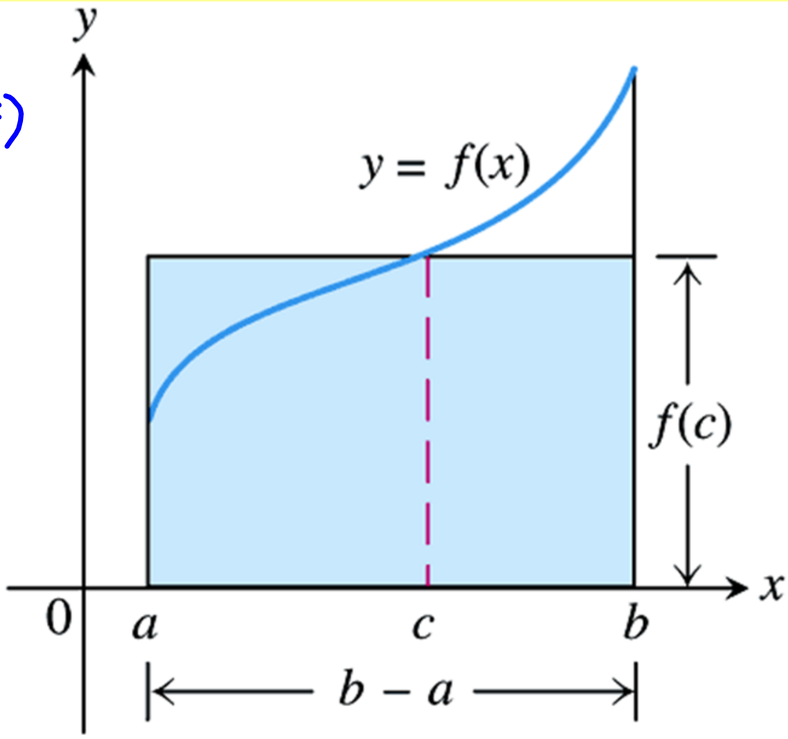
$$=-\frac{10}{3}$$

The Mean Value Theorem for Definite Integrals

If f is continuous on $[a,b]$, then at some point c in $[a,b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx.$$

avg (f)



The Derivative of an Integral

$$\frac{d}{dx}\int_a^x f(t)dt = f(x).$$

EXAMPLES:

Together

$$\frac{d}{dx}\int_4^x t^2\,dt = f(x) = x^2$$

$$\frac{d}{dx}\int_\pi^x \ln t\,dt = f(x) = \ln x$$

$$\frac{d}{dx}\int_1^{x^2} e^t\,dt = f(x) = e^{x^2}$$

$$\frac{d}{dx}\int_4^{3x} \cos t\,dt = f(x) = \cos(3x)$$

On your own

$$\frac{d}{dx}\int_{-1}^x \cos t\,dt = \cos x$$

$$\frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\int_4^{2x} \frac{1}{1+t^2}\,dt = \frac{1}{1+(2x)^2}$$

or $\frac{1}{1+4x^2}$

$$\frac{d}{dx}\int_{12}^{3x^2-x} \sin^2 t\,dt$$

$= \sin^2(3x^2-x)$

Antiderivatives

$$\int_a^x f(t)\,dt = F(x) - F(a)$$ (where F is antiderivative of f.)

Find $\int_0^{\pi} \sin x \, dx$ using

Since $\sin x$ is the rate of change of $F(x) = -\cos x$, meaning $F'(x) = \sin x$.

$$\int_0^{\pi} \sin x \, dx = -\cos(\pi) - (-\cos(0)) =$$

$$-(-1) - (-1) = \boxed{2}$$

Homework

6.3: pg.294-5 #1, 2, 5, 19-22, 32, 33