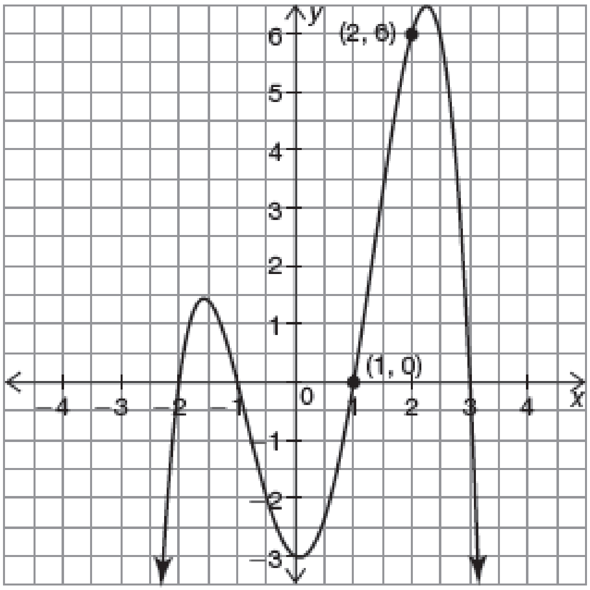
Questions on Lesson 6.1?

No quiz today, but work on finding the **average rate of change** for the following function, given the interval below.

(1, 2)



$$\frac{6-0}{2-1}=\frac{6}{1}=6$$

x x 3 1

62) 1976.00

-186 ↓

116

- 62

54

31 R54

$31\frac{54}{62}$

1 0 6

^{2}24) 2567

-24 ↓ ↓

167

-144

23

106 R23

$106\frac{23}{24}$

The Great Polynomial Divide

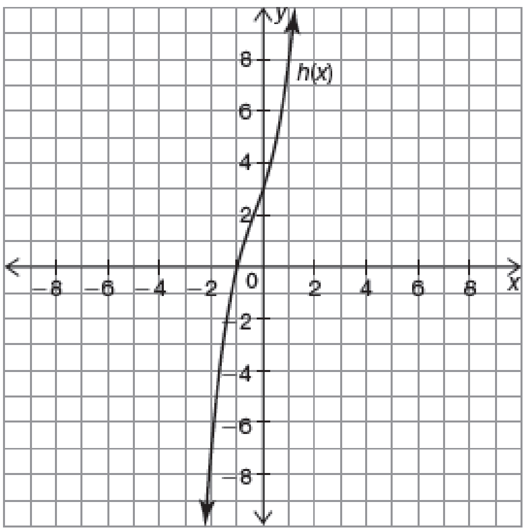
Polynomial Division

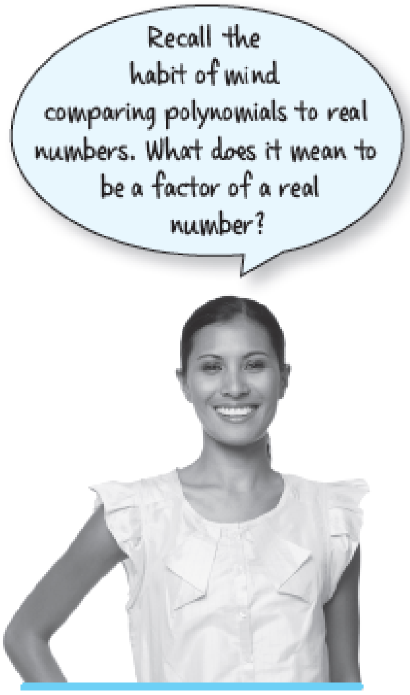
6.2

pg.433-434 in your book

The previous function-building lessons showed how the factors of a polynomial determine its key characteristics. From the factors, you can determine the type and location of a polynomial's zeros. Algebraic reasoning often allows you to reverse processes and work backwards. Specifically in this problem, you will determine the factors from one or more zeros of a polynomial from a graph.

1. Analyze the graph of the function $h(x) = x^3 + x^2 + 3x + 3$.





a. Describe the number and types of zeros of $h(x)$.

3; 1 real #, 2 imaginary

$$2\overline{)8}^{\,4}$$
-8
0

b. Write the factor of $h(x)$ that corresponds to the zero at $x = -1$.

$(x+1)$

$+1 \quad +1$
$x+1=0$

c. What does it mean to be a factor of $h(x) = x^3 + x^2 + 3x + 3$?

When we divide h(x) by the factor, we get a remainder of 0.

d. How can you write any zero, r, of a function as a factor?

$(x-r)$

pg.437 in your book.

The Fundamental Theorem of Algebra states that every polynomial equation of degree *n* must have *n* roots. This means that every polynomial can be written as the product of *n* factors of the form $(ax + b)$. For example, $2x^2 - 3x - 9 = (2x + 3)(x - 3)$.

→ linear

You know that a factor of an integer divides into that integer with a remainder of zero. This process can also help determine other factors. For example, knowing 5 is a factor of 115, you can determine that 23 is also a factor since $\frac{115}{5} = 23$. In the same manner, factors of polynomials also divide into a polynomial without a remainder. Recall that $a \div b$ is $\frac{a}{b}$, where $b \neq 0$.

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. The process is similar to integer long division.

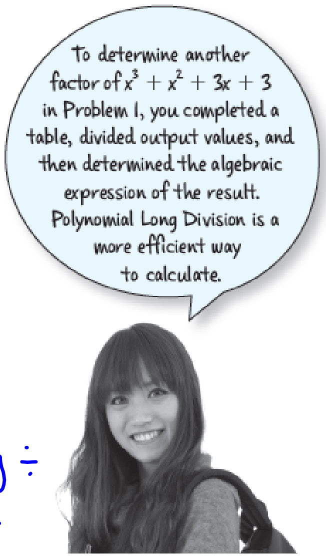
Notice in the dividend of the polynomial example, there is a gap in the degrees of the terms; every power must have a placeholder. The polynomial $8x^3 - 12x - 7$ does not have an x^2 term.

Integer Long Division	Polynomial Long Division	Description
4027 ÷ 12 or $\frac{4027}{12}$	$(8x^3 - 12x - 7) \div (2x + 3)$ or $\frac{8x^3 - 12x - 7}{2x + 3}$	
335 $12\overline{)4027}$ −36 42 −36 67 −60 ⑦ Remainder	Ⓑ $4x^2 -$ Ⓔ $6x +$ Ⓗ 3 $2x + 3\overline{)}$ Ⓒ $8x^3 +$ Ⓐ $0x^2 - 12x - 7$ $-(8x^3 + 12x^2)$ Ⓕ $-12x^2 - 12x$ Ⓓ $-(-12x^2 - 18x)$ Ⓘ $6x - 7$ Ⓖ $-(6x + 9)$ Remainder −16	A. Rewrite the dividend so that each power is represented. Insert $0x^2$. B. Divide $\frac{8x^3}{2x} = 4x^2$. C. Multiply $4x^2(2x + 3)$, and then subtract. D. Bring down $-12x$. E. Divide $\frac{-12x^2}{2x} = -6x$. F. Multiply $-6x(2x + 3)$, and then subtract. G. Bring down -7. H. Divide $\frac{6x}{2x} = 3$. I. Multiply $3(2x + 3)$, and then subtract.
$\frac{4027}{12} = 335$ R 7	$\frac{8x^3 - 12x - 7}{2x + 3} =$ $4x^2 - 6x + 3$ R -16	Rewrite
$4027 = (12)(335) + 7$	$8x^3 - 12x - 7 =$ $(2x + 3)(4x^2 - 6x + 3) - 16$	Check

pg.438 in your book.

1. Analyze the worked example that shows integer long division and polynomial long division.

   a. In what ways are the integer and polynomial long division algorithms similar?



   b. Is $2x + 3$ a factor of $f(x) = 8x^3 - 12x - 7$? Explain your reasoning.

   No, because when we long ÷ the remainder is not 0.

2. Determine the quotient for each. Show all of your work.

$1x \cdot 4x^2 = 4x^3$ $\quad \frac{4x^3}{x} = 4x^2$

a. $x\overline{)4x^3 - 0x^2 + 7x}$

$4x^2 + 0x + 7$

$- 4x^3$

$0 - 0x^2$

$-0x^2$

$0x^2 + 7x$

$- 7x$

0

$4x^3 + 7x = x(4x^2 + 7)$

Answer: $4x^2 + 7$

$x \cdot \_ = x^3$

$\frac{x^3}{x} = x^2$

$x \cdot 6x = 6x^2$

b. $x - 4\overline{)x^3 + 2x^2 - 5x + 16}$

$x^2 + 6x + 19$

$-(\cancel{x^3} - 4x^2)$

$\cancel{6x^2} - 5x + 16$

$-(\cancel{6x^2} - 24x)$

$\cancel{19x} + 16$

$-(\cancel{19x} - 76)$

$\boxed{92}$

Answer: $x^2 + 6x + 19 + \frac{92}{x-4}$

c. $(4x^4 + 5x^2 - 7x + 9) \div (2x - 3)$

*place holder x^3

$2x^3 + 3x^2 + 7x + 7$

$2x-3\overline{)4x^4 + 0x^3 + 5x^2 - 7x + 9}$

$-(\cancel{4x^4} - 6x^3)$

$\cancel{6x^3} + 5x^2 - 7x + 9$

$-(\cancel{6x^3} - 9x^2)$

$\cancel{14x^2} - 7x + 9$

$-(\cancel{14x^2} - 21x)$

$\cancel{14x} + 9$

$-(\cancel{14x} - 21)$

$\boxed{30}$

Answer: $2x^3 + 3x^2 + 7x + 7 + \frac{30}{2x-3}$

d. $(9x^4 + 3x^3 + 4x^2 + 7x + 2) \div (3x + 2)$

$3x^3 - 1x^2 + 2x + 1$

$3x+2\overline{)9x^4 + 3x^3 + 4x^2 + 7x + 2}$

$-(\cancel{9x^4} + 6x^3)$

$-\cancel{3x^3} + 4x^2 + 7x + 2$

$-(-\cancel{3x^3} - 2x^2)$

$\cancel{6x^2} + 7x + 2$

$-(\cancel{6x^2} + 4x)$

$3x + 2$

$-(3x + 2)$

$\boxed{0}$

Answer: $3x^3 - x^2 + 2x + 1$

HW: pg 439-443

pg.441 in your book.

6. Calculate the quotient using long division. Then write the dividend as the product of the divisor and the quotient plus the remainder.

a. $f(x) = x^2 - 1$

$g(x) = x - 1$

Calculate $\frac{f(x)}{g(x)}$.

b. $f(x) = x^3 - 1$

$g(x) = x - 1$

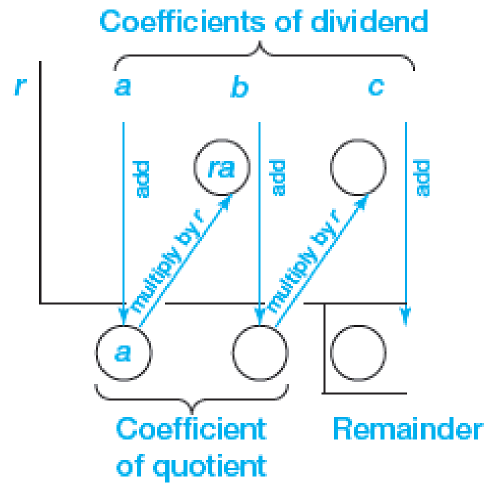
Calculate $\frac{f(x)}{g(x)}$.

pg.443 in your book.

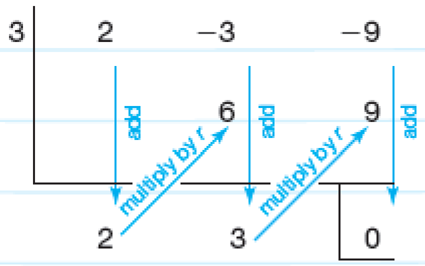
Although dividing polynomials through long division is analogous to integer long division, it can still be inefficient and time consuming. **Synthetic division** is a shortcut method for dividing a polynomial by a linear factor of the form ($x - r$). This method requires fewer calculations and less writing by representing the polynomial and the linear factor as a set of numeric values. After the values are processed, you can then use the numeric outputs to construct the quotient and the remainder.

pg.444 in your book.

To use synthetic division to divide a polynomial $ax^2 + bx + c$ by a linear factor $x - r$, follow this pattern.



You can use synthetic division in place of the standard long division algorithm to determine the quotient for $(2x^2 - 3x - 9) \div (x - 3)$.

Long Division	**Synthetic Division**
$\begin{array}{r} 2x+3 \\ x-3\overline{)2x^2-3x-9} \\ \underline{2x^2-6x} \\ 3x-9 \\ \underline{3x-9} \\ 0 \end{array}$	$r = 3$ 
$(2x^2 - 3x - 9) \div (x - 3) = 2x + 3$	

pg.445-6 in your book.

2. Two examples of synthetic division are provided. Perform the steps outlined for each problem:

i. Write the dividend.

ii. Write the divisor.

iii. Write the quotient.

iv. Write the dividend as the product of the divisor and the quotient plus the remainder.

a.

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -4 & -3 & 6 \\ & & 2 & 4 & 0 & -6 \\ \hline & 1 & 2 & 0 & -3 & \boxed{0} \end{array}$$

i.

ii.

iii.

iv.

3. Calculate each quotient using synthetic division. Then write the dividend as the product of the divisor and the quotient plus the remainder.

a. $g(x) = x^3 + 1$

$r(x) = x + 1$

Calculate $\frac{g(x)}{r(x)}$.

not in your book.

1. The given table of values represents the function $f(x) = x^3 + 9x^2 + 14x - 24$.

x	−2	−1	0	1	2
$f(x)$	−24	−30	−24	0	48

a. Determine one of the factors of $f(x)$ without using a calculator. Explain your reasoning.

b. Completely factor $f(x)$ without using a calculator.

c. Determine all of the zeros of $f(x)$ without using a calculator.

2. Determine whether $2x - 4$ is a factor of $m(x) = 2x^4 - 8x^2 + 4$.

not in your book.

3. The Polynomial Pool Company offers 10 different pool designs numbered 1 through 10. Each pool is in the shape of a rectangular prism. The volume of water in Pool Design x, can be determined using the function $V(x) = \ell(x) \cdot w(x) \cdot d(x) = 2x^3 + 18x^2 + 46x + 30$, where $\ell(x)$, $w(x)$, and $d(x)$ represent the length, width, and depth of the pool in feet.
   a. Determine the expressions for the functions $w(x)$ and $d(x)$ if $\ell(x) = 2x + 2$ and the width of each pool is greater than the depth. Do not use a calculator.

   b. Determine the length, width, and depth of Pool Design 9.

4. Determine whether $3x + 3$ is a factor of $p(x) = 3x^4 + 3x^3 - 6x^2 - 6x$.

Nov 19-8:37 AM

Homework

Finish Lesson 6.2