Friday, January 13 is the last day Ms. Hansen will accept any late/missing/extra credit work for 2nd quarter

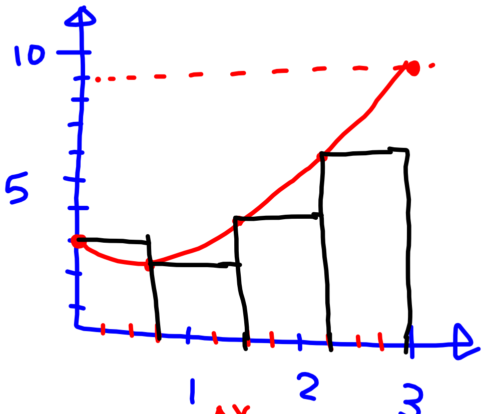
-->This includes any test/quiz make ups.

Questions on 6.1 HW?

9 & 12

9) $f(x) = x^2 - x + 3$, $0 \le x \le 3$

* LRAM
4 subintervals
$\Delta x = 3/4$



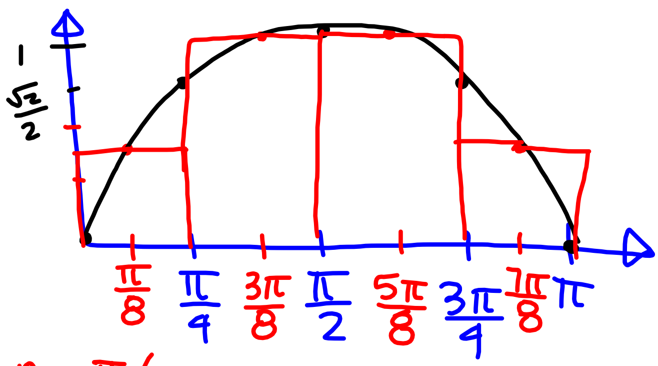
x	$f(x)$
0	3
3/4	2.8125
1.5	3.75
2.25	5.8125
3	9

$$A = \underbrace{\frac{3}{4}}_{\Delta x}\left(\underbrace{3}_{f(c_1)} + \underbrace{2.8}_{f(c_2)} + \underbrace{3.8}_{f(c_3)} + \underbrace{5.8}_{f(c_4)}\right)$$

$A = 11.55$

(12) $f(x) = \sin x$, $0 \le x \le \pi$

* MRAM, 4 subintervals
$\Delta x = \frac{\pi}{4}$



x	$f(x)$
0	0
$\pi/8$	0.38
$\pi/4$	$\sqrt{2}/2$
$3\pi/8$	0.92
$\pi/2$	1
$5\pi/8$	0.92
$3\pi/4$	$\sqrt{2}/2$
$7\pi/8$	0.34
π	0

$$A = \frac{\pi}{4}(0.38 + 0.92 + 0.92 + 0.38)$$

$A = 2.278$

6.2 Definite Integrals

Sigma Notation

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$$

first term

$$\underbrace{3}_{a_1} + \underbrace{5}_{a_2} + \underbrace{7}_{a_3} + \underbrace{9}_{a_4} + \underbrace{11}_{a_5}$$

The Definite Integral as a Limit of Reimann Sums (FYI - do not copy down)

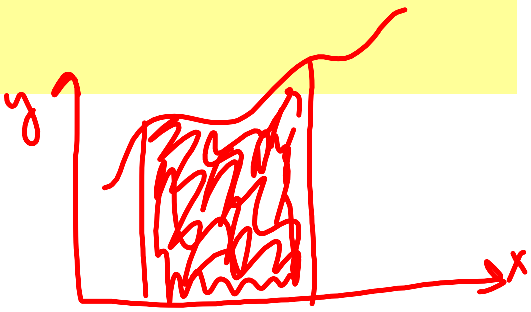
Let f be a function defined on a closed interval $[a,b]$. For any partition P of $[a,b]$, let the numbers c_k be chosen arbitrarily in the subinterval $[x_{k-1}, x_k]$.

If there exists a number I such that $\lim\limits_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k = I$

no matter how P and the c_k's are chosen, then f is **integrable** on $[a,b]$ and I is the **definite integral** of f over $[a,b]$.

The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a,b]$, then its definite integral over $[a,b]$ exists.



The Definite Integral of a Continuous Function on [a,b]

Let f be continuous on $[a,b]$, and let $[a,b]$ be partitioned into n subintervals of equal length $\Delta x = (b-a)/n$. Then the definite integral of f over $[a,b]$ is given by $\lim_{n\to\infty}\sum_{k=1}^{n} \overbrace{f(c_k)}^{y}\Delta x$, where each c_k is chosen arbitrarily in the k^{th} subinterval.

$\int$

The Definite Integral (Common Notation)

$$\underbrace{\int_a^b f(x)\,dx}$$

upper limit of integration

integral sign

lower limit of integration

function (f(x)) is integrand

x is the variable of integration

Integral of f from a to b; when you find the value of the integral, you have evaluated the integral.

Example

The interval $[-2, 4]$ is partitioned into n subintervals of equal length $\Delta x = 6/n$. Let m_k denote the midpoint of the k^{th} subinterval. Express the limit

$\lim_{n\to\infty}\sum_{k=1}^{n}\left(3(m_k)^2 - 2m_k + 5\right)\Delta x$ as an integral.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(3(m_k)^2 - 2m_k + 5\right)\Delta x = \int_{-2}^{4}\left(3x^2 - 2x + 5\right)dx$$

Answer

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(3(m_k)^2 - 2m_k + 5\right)\Delta x = \int_{-2}^{4}\left(3x^2 - 2x + 5\right)dx$$

Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a,b]$, then the area under the curve $y = f(x)$ from a to b is the **integral of f from a to b**, $A = \int_a^b f(x)dx$.

Area

negative only

$$\text{Area} = -\int_a^b f(x)dx \quad \text{when } f(x) \le 0.$$

** negative & positive

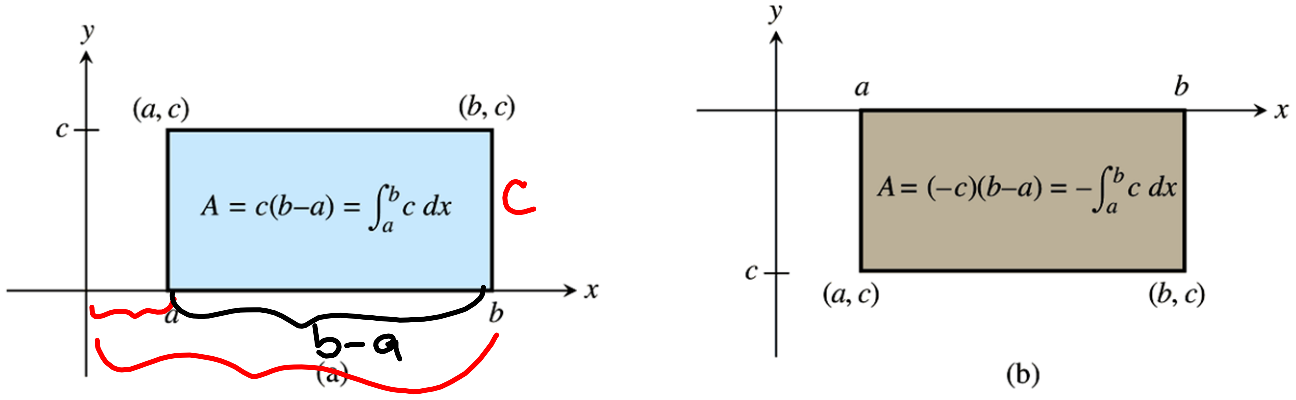
$$\int_a^b f(x)dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

**If an integrable function y = f(x) has both positive and negative values on an interval [a,b], then the Riemann sums for f on [a,b] add areas of rectangles that lie above the x-axis to the negatives of areas of rectangles that lie below the x-axis. The resulting cancellations mean that the limiting value is a number whose mangitude is less than the total area between the curve and the x-axis. The value of the integral is the area above the x-axis minus the area below.

The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a, b]$, then

$$\int_a^b f(x)dx = \int_a^b c\,dx = c(b-a)$$



Example using NINT

$$\int_a^b f(x)dx = \text{NINT}(f(x), x, a, b)$$

Evaluate numerically. $\int_{-1}^{2} x\sin x\,dx$

$$\text{NINT}(x\sin x, x, -1, 2) \approx 2.04$$

Jan 9-2:15 PM

EXAMPLES:

Together

$$\frac{d}{dx}\int_4^x t^2\,dt =$$

$$\frac{d}{dx}\int_\pi^x \ln t\,dt$$

$$\frac{d}{dx}\int_1^{x^2} e^t\,dt$$

$$\frac{d}{dx}\int_4^{3x} \cos t\,dt$$

On your own

$$\frac{d}{dx}\int_{-1}^x \cos t\,dt$$

$$\frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt$$

$$\frac{d}{dx}\int_4^{2x} \frac{1}{1+t^2}\,dt$$

$$\frac{d}{dx}\int_{12}^{3x^2-x} \sin^2 t\,dt$$

Homework

6.2: pg.286-7 #1-12, 13, 14

Section 6.2 Exercises

In Exercises 1–6, each c_k is chosen from the kth subinterval of a regular partition of the indicated interval into n subintervals of length Δx. Express the limit as a definite integral.

1. $\lim_{n\to\infty} \sum_{k=1}^{n} c_k^2 \Delta x$, $[0, 2]$

2. $\lim_{n\to\infty} \sum_{k=1}^{n} (c_k^2 - 3c_k)\Delta x$, $[-7, 5]$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{c_k} \Delta x$, $[1, 4]$

4. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1 - c_k} \Delta x$, $[2, 3]$

5. $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{4 - c_k^2}\, \Delta x$, $[0, 1]$

6. $\lim_{n\to\infty} \sum_{k=1}^{n} (\sin^3 c_k)\, \Delta x$, $[-\pi, \pi]$

In Exercises 7–12, evaluate the integral.

7. $\int_{-2}^{1} 5\, dx$

8. $\int_{3}^{7} (-20)\, dx$

9. $\int_{0}^{3} (-160)\, dt$

10. $\int_{-4}^{-1} \frac{\pi}{2}\, d\theta$

11. $\int_{-2.1}^{3.4} 0.5\, ds$

12. $\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\, dr$

In Exercises 13–22, use the graph of the integrand and areas to evaluate the integral.

13. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

14. $\int_{1/2}^{3/2} (-2x + 4)\, dx$

15. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

16. $\int_{-4}^{0} \sqrt{16 - x^2}\, dx$

17. $\int_{-2}^{1} |x|\, dx$

18. $\int_{-1}^{1} (1 - |x|)\, dx$

19. $\int_{-1}^{1} (2 - |x|)\, dx$

20. $\int_{-1}^{1} (1 + \sqrt{1 - x^2})\, dx$

21. $\int_{\pi}^{2\pi} \theta\, d\theta$

22. $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

Oct 21-8:47 AM